SIMULATION OF THREE-DIMENSIONAL GRANULAR DISPLACEMENT IN UNCONSOLIDATED AQUIFERS

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Hydrodynamic processes associated with land subsidence and subsequent earth fissuring due to fluid withdrawal in unconsolidated aquifers are three dimensional in scope. Mathematical and numerical models that use hydraulic head or volume strain as the principal unknown variable are traditionally one dimensional with respect to changes in storage and strain. These models can simulate the total vertical compaction of interbeds in a confined aquifer, but they have no way of predicting directional components of granular movement or the resulting strain field. Consequently, they cannot estimate where damaging fissures may occur over time. For research purposes a new three-dimensional numerical model is being developed that is based on the modular finite-difference ground-water flow model (MODFLOW) by McDonald and Harbaugh (1988) that has the displacement field of solids as its principal unknown variable (see Leake abstract for the current status of MODFLOW and packages for simulating land subsidence). The governing equation (Helm, 1987) can be written as:

$$\frac{d}{dt}\vec{u} - \frac{\bar{K}}{\rho_w g \alpha}\nabla(\nabla \bullet \vec{u}) = \vec{q}_b - \vec{q}_o + \frac{\bar{K}}{\rho_w g}\nabla\sigma_m$$

where $\vec{u}$ is the displacement of solids, K is the hydraulic conductivity tensor, ρ_w is the density of water, g is the gravity constant, α is the compressibility of the skeletal matrix, $\vec{q}_b$ is the bulk flux, $\vec{q}_o$ is the initial unstrained specific discharge, and σ_m is the mean total stress. Because the displacement field of solids is a vector quantity, granular displacement resulting from imposed stresses on an unconsolidated aquifer can be simulated in three dimensions. The new model is not limited to confined aquifers, but can readily be applied to unconfined and semiconfined aquifers.

The three-dimensional governing equation used in the new model inherently assumes that ground-water flow is relative to solids. Thus, a new set of initial conditions is needed to account for the solid matrix. This quantity is the bulk flux that takes into account both the velocity of water and the velocity of solids (see Helm abstract). Expressions are developed for the bulk flux for both a pumping well (or artificial recharge well) and natural recharge by infiltration of precipitation.

The general three-dimensional form of the governing equation is difficult to apply numerically and does not comply to the general structure of the modular ground-water flow model. A more tractable and simpler approach is to uncouple the governing equation into three one- dimensional expressions. This was accomplished by assuming that within a specified material the changes or gradients of shear strain are small in comparison to the changes or gradients of normal strain in the principal directions. In this way the

displacement of solids can be viewed as a scalar quantity, much like hydraulic head in the ground-water flow equation. The result is a separate diffusion-style equation for each component direction as follows:

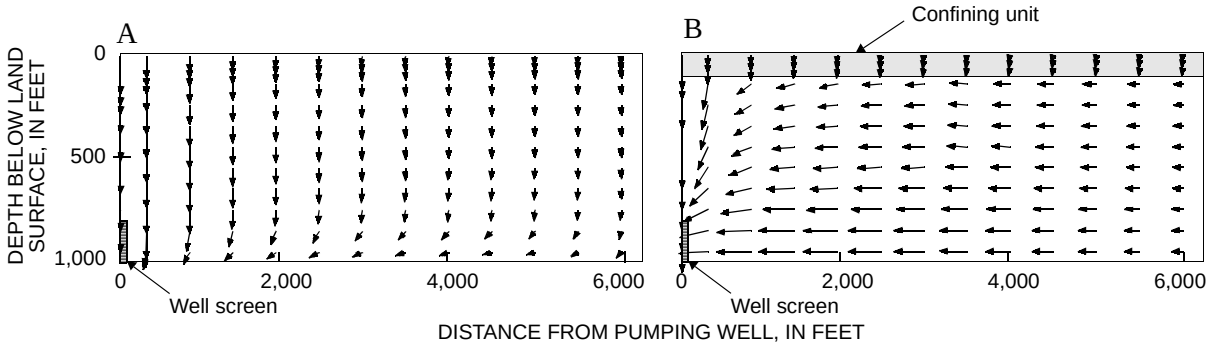
$$\frac{du_x}{dt} - \frac{K_{xx}}{3\rho_w g \alpha_{xx}}\left(\frac{\partial^2 u_x}{\partial x^2}\right) = q_{bx} - q_{ox} \ ,$$

$$\frac{du_y}{dt} - \frac{K_{yy}}{3\rho_w g \alpha_{yy}}\left(\frac{\partial^2 u_y}{\partial y^2}\right) = q_{by} - q_{oy} \ ,$$

$$\frac{du_z}{dt} - \frac{K_{zz}}{3\rho_w g \alpha_{zz}}\left(\frac{\partial^2 u_z}{\partial z^2}\right) = q_{bz} - q_{oz} \ ,$$

where the subscripts x, y, and z represent the principal directions. A Crank-Nicolson numerical procedure, which is second-order correct and unconditionally stable, is used to approximate the resulting three one-dimensional governing equations. The uncoupled expressions for each component direction form a tridiagonal matrix that can be solved directly. Boundary conditions are developed for both zero displacement and impermeable lateral boundaries. A zero volume-strain rate condition is used to simulate the location of the water table. The aquifer bottom, representing basement rock, is always assumed to be a zero-displacement boundary.

Hypothetical simulations are developed for unconfined isotropic, unconfined anisotropic, and confined isotropic aquifers. A 12-layered 1,000-ft-thick system with a single pumping well having a 100-ft-thick screened interval is used to test the new model. Simulation results for the isotropic condition indicate that displacement is symmetrical and inward toward the pumping well. The spherical radius of maximum inward displacement progresses outward from the well as simulation time increases. The shape of this “radius” becomes modified by boundary conditions such as the water table. Thus, for a short simulation time, the maximum downward vertical displacement is not at land surface but rather at a point nearer the screened interval of the pumping well. In fact, a poisson type of effect occurs as small upward displacements at the water table are simulated in a donut-shaped pattern around the well bore. For the anisotropic condition, when vertical hydraulic conductivity becomes small relative to horizontal hydraulic conductivity, the displacement field changes from mostly horizontal to mostly vertical. A dominant downward displacement is simulated in all layers above the screened interval for strongly anisotropic conditions (fig. 1A) The lateral extent at which a dominant downward component of displacement occurs depends on the degree of anisotropy. For the confined condition, displacement is primarily downward in the confining unit but is mostly horizontal within the homogeneous isotropic aquifer beneath the confining unit (fig. 1B). Near the well, vertical displacements become significant within the aquifer also. Within the region of horizontal compression in radially extensive confined aquifers, the displacement model simulates less vertical subsidence than conventional models that ignore horizontal granular movement.



Aquifer Condition or Property	Anisotropic (fig. 1A)	Confined (fig. 1B)
Horizontal hydraulic conductivity of confining unit	not applicable	1.0 ft/day
Vertical hydraulic conductivity of confining unit	not applicable	0.0001 ft/day
Horizontal hydraulic conductivity of aquifer	10 ft/day	10 ft/day
Vertical hydraulic conductivity of aquifer	0.01 ft/day	10 ft/day
Elastic specific storage of confining unit	not applicable	2×10^{-5} foot^{-1}
Inelastic specific storage of confining unit	not applicable	8×10^{-4} foot^{-1}
Elastic specific storage of aquifer	2×10^{-6} foot^{-1}	2×10^{-6} foot^{-1}
Inelastic specific storage of aquifer	8×10^{-6} foot^{-1}	8×10^{-6} foot^{-1}
Maximum horizontal displacement	0.0073 foot	0.0261 foot
Maximum vertical displacement	0.3809 foot	0.0866 foot
Pumping rate	2.0 ft^3/s	2.0 ft^3/s
Total pumping time	500 days	500 days

Figure 1. Granular displacement in (A) an unconfined isotropic aquifer and in (B) a confined isotropic aquifer, with accompanying table describing values used for each simulation. Vector arrows indicate relative magnitude of displacement.