State of Subsidence Modeling Within the U.S. Geological Survey

By Thomas J. Burbey

Much of the current knowledge of land subsidence and aquifer deformation came from the U.S. Geological Survey's Mechanics of Aquifers Project led by Joseph F. Poland from 1955 through 1984. Until the advent of efficient and powerful computers, many of the calculations used to estimate vertical subsidence were done with analytical models (Poland and Davis, 1969). Beginning in the 1970's, Terzaghi's principle of effective stress coupled with Hubbert's force potential and Darcy's Law provided the basis for one-dimensional subsidence modeling (Gambolati and others, 1974; Helm, 1975, 1976). Helm's one-dimensional consolidation model was developed for constant and stress-dependent parameters, but was not linked to a ground-water flow model. Although Helm's model remains a powerful tool for detailed analysis of vertical effects at a specific site, it is not a model for basin-wide analysis. The first subsidence model incorporated into a ground-water flow model was written by Meyer and Carr (1979). This model allowed for elastic- and inelastic-storage values to be incorporated into a three-dimensional ground-water flow model (Trescott, 1975). After the development of the three-dimensional MODFLOW ground-water flow model (McDonald and Harbaugh, 1988), Leake and Prudic (1991) wrote a one-dimensional subsidence program for MODFLOW called the Interbed Storage Package (IBS1). This subsidence model is more versatile than the Meyer and Carr (1979) model and is used today as the standard for modeling subsidence in ground-water basins. Although this program does not contain the stress-dependent parameter capabilities of the earlier Helm model, it allows for continuous calculation of subsidence due to pumping in the areal extent of the model grid. Leake (1990) added other capabilities to the original code by allowing the evaluation of delayed drainage from interbeds within an aquifer system in an experimental version of the Interbed Storage Package (IBS2). Leake (1991) developed another Interbed Storage Package (IBS3) in which total load can be treated as a variable and storage parameters as stress dependent. These models can be used to evaluate vertical subsidence due to fluid withdrawal; however, they do not account for horizontal displacement resulting from changes in stress.

The surface effects of horizontal displacement have been evaluated by measuring radial strains (Wolff, 1970), by observance of failed surface structures (Poland and Davis, 1969) and through the presence of earth fissures (Holzer, 1984). Early explanations have associated the occurrence of horizontal movement with differential subsidence and compared the process to that of a bending beam failing at the point of greatest stress. That is, horizontal movement occurs primarily above the aquifer in the brittle unsaturated zone and is a direct consequence of vertical displacement. This theory does not apply to many fissures and structural failures that have been observed where minimal subsidence has been measured (Holzer, 1984; Anderson, 1989). In recent years, earth fissures have been shown to be directly related to horizontal aquifer movement due to pumping in unconsolidated aquifers (Helm, 1994a). In addition, many fissures are known to have migrated upwards from depth thus contradicting the earlier bending-beam theory of fissure development. Theoretical developments (Bear and Corapcioglu, 1981; Helm, 1994b) and field measurements of horizontal land-surface movement (Poland and Davis, 1969) indicate that horizontal aquifer movement is significant and can be the same order of magnitude as vertical subsidence. Furthermore, Helm (1994b) indicates that horizontal movement due to pumping can occur beyond where measurable drawdown or subsidence occurs. Thus, the problems associated with land subsidence are three dimensional in scope. The calculation of both vertical and horizontal movement provides the necessary information water managers need to optimize pumping and reduce the potential for earth-fissure development. Earth fissures have resulted in many litigation cases because of structural damage caused to buildings, foundations, fences, railroads, roads, sidewalks, pipelines, and well casings. The next step in advancing the state of subsidence modeling is to include the mathematics needed to produce a model capable of simulating both horizontal and vertical aquifer-system deformation.

Development of a tractable field-based three-dimensional displacement model to simulate aquifer-system response to changes in applied stress is the next goal. Earlier subsidence models cannot provide the foundation for a three-dimensional displacement model because three-dimensional poroelastic theory is different from the theory used in one-dimensional subsidence models that are based solely on stress changes due to water-level declines. In Biot's (1941, 1955) development of three-dimensional consolidation (poroelastic) theory, the directional components of displacement and pressure or hydraulic head are dependent variables. This development incorporates the principle of effective stress and inherently assumes stress equilibrium and an elastic stress-strain constitutive relation. The resulting governing equation can be expressed as

$$(\lambda + G)\nabla(\nabla \bullet \boldsymbol{u}_s) + G\nabla^2 \boldsymbol{u}_s = \rho_w g \nabla h \ , \qquad (3)$$

where ρ_w is the density of water, g is the gravitational constant, h is hydraulic head, $\boldsymbol{u}_s$ is the displacement field of solids, G is the shear modulus (and one of Lamé's constants), and λ is the other Lamé constant. Lamé's constant λ is defined in terms of the shear modulus G and Poisson's ratio ν as

$$\lambda = \frac{2Gn}{1 - 2\nu} , \qquad (4)$$

Equation (1) represents a system of three equations with four unknowns. Another equation is needed that relates hydraulic head to the displacement field of solids. The fourth equation is obtained by first writing Darcy's Law in terms of the velocity of solids,

$$n(\upsilon_w - \upsilon_s) = -\kappa \nabla h \ , \qquad (5)$$

where υ_w is the velocity of water, υ_s is the velocity of solids, κ is hydraulic conductivity, and n is porosity. Assuming constant water and solid-grain density, applying the principle of conservation of fluid and solid mass, taking the divergence of all the terms of equation (3), and relating volume strain to displacement yields the fourth equation,

$$\frac{\partial}{\partial t}(\nabla \bullet \boldsymbol{u}_s) = \kappa \nabla^2 h \ . \qquad (6)$$

Equations (1) and (4) are Biot's fundamental expressions of consolidation and have been used by P.A. Hsieh (see paper entitled "Poroelasticity Simulation of Ground-Water Flow and Subsurface Deformation, p. 5, this report) for developing a two-dimensional axisymmetric finite-element displacement model (referred to in this report as HDM).

Capabilities for simulating three-dimensional poroelasticity combined with the power and flexibility of MODFLOW would result in a valuable tool for analysis of aquifer deformation. Equation (4), however, is not compatible with MODFLOW, which uses specific storage instead of displacement or strain. Rice and Cleary (1976) use an alternative formulation of poroelastic theory. Their governing equation can be expressed as

$$\frac{\partial h}{\partial t} - \kappa\left(\frac{3\lambda + 2G}{3\rho_w g}\right)\nabla^2 h = \frac{1}{\rho_w g}\ \frac{\partial(\delta s_m)}{\partial t} \ , \qquad (7)$$

where $\rho\sigma_m$ is the incremental change in mean total stress. Skeletal specific storage (S_s) for three-dimensional problems is defined as

$$S_s = \frac{3\rho_w g}{3\lambda + 2G} . \qquad (8)$$

Where and when the change in mean total stress appropriately is assumed to be negligible within a ground-water basin, equation (5) becomes identical to the traditional ground-water flow equation used in MODFLOW and other ground-water flow models. Assuming the change in mean total stress is negligible may eliminate the ability to simulate reversals of direction of change in water levels (Noordbergum effect) frequently observed during early times of pumping. None- theless, using this traditional expression with equation (1) results in a powerful coupling of equations to simulate three-dimensional consoli- dation (referred to here as the granular displace- ment model, or GDM) and three-dimensional ground-water flow within MODFLOW. Because basin-wide subsidence studies generally involve long time periods (simulation time of decades with individual time steps of a month or more), the change in mean total stress and the occurrence of reversals of direction of change in water levels probably are less significant. Neglecting the change in mean total stress, therefore, may not significantly affect results for long-term basin-wide simulations.

To evaluate the legitimacy of this approach, the HDM and GDM models are compared for three periods assuming an isotropic, confined-aquifer system with zero-displacement boundaries along the bottom and sides and a zero-traction boundary (zero total load) at the aquifer top. The side or lateral boundaries are more than 10,000 m from the pumping well and do not affect simulation results. Aquifer properties and initial conditions used in the simulations are shown in table 1. The calculated vertical and horizontal displacements resulting from the two models are shown in figures 1 and 2. Results indicate that for modeling aquifer-system displacements due to fluid withdrawal, the change in mean total stress may not be large, even for short time steps. Results indicate that the small, simulated differences in horizontal displacement may be due to the different numerical schemes or coordinate systems used in the two models. The GDM is an improvement over other models because it offers the power and flexibility of MODFLOW with the ability to simulate aquifer-system deformation in three dimensions. This model will help provide a better understanding of location and severity of potentially damaging fissures.

REFERENCES CITED

Anderson, S.R., 1989, Potential for aquifer compaction, land subsidence, and earth fissures in Avra Valley, Pima and Pinal Counties, Arizona: U.S. Geological Survey Hydrologic Investigations Atlas HA–718, 3 sheets, scale 1:250,000.

Bear, Jacob, and Corapcioglu, M.Y., 1981, Mathematical model for regional land subsidence due to pumping—2. Integrated aquifer subsidence equations for vertical and horizontal displacements: American Geophysical Union, Water Resources Research, v. 17, no. 4, p. 947–958.

Biot, M.A., 1941, General theory of three-dimensional consolidation: Journal of Applied Physics, v. 12, p. 155–164.

—— 1955, Theory of elasticity and consolidation for a porous anisotropic solid: Journal of Applied Physics, v. 26, p. 182–185.

Gambolati, Giuseppe, Gatto, Paolo, and Freeze, R.A., 1974, Mathematical simulation of the subsidence of Venice—2. Results: American Geophysical Union, Water Resources Research, v. 10, no. 3, p. 563–577.

Helm, D.C., 1975, One-dimensional simulation of aquifer system compaction near Pixley,

Table 1. Aquifer properties and initial conditions used in the simulations to produce the results shown in figures 1 and 2

Property or condition	Value
Pumping rate, *Q*, in cubic meters per second	6.3×10^{-3}
Hydraulic conductivity, κ, in meters per second	1×10^{-4}
Drained Poisson's ratio, ν, dimensionless	.25
Shear modulus, *G*, in Newton per square meter	6.533333×10^{6}
Specific weight of water, $\rho_w g$, in Newton per cubic meter	9.8×10^{3}
Lamé's constant, λ, in Newton per square meter	6.533333×10^{6}

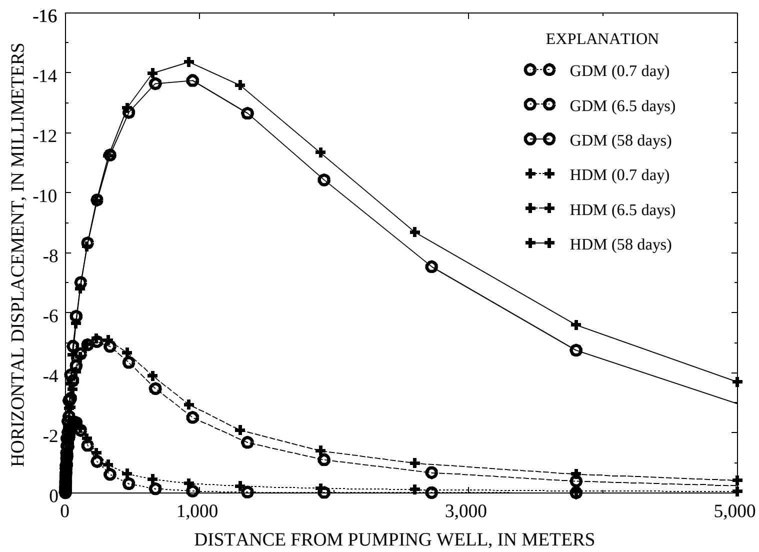

Figure 1. Simulated horizontal displacement for three time periods using the granular displacement model (GDM) and the Hsieh displacement model (HDM).

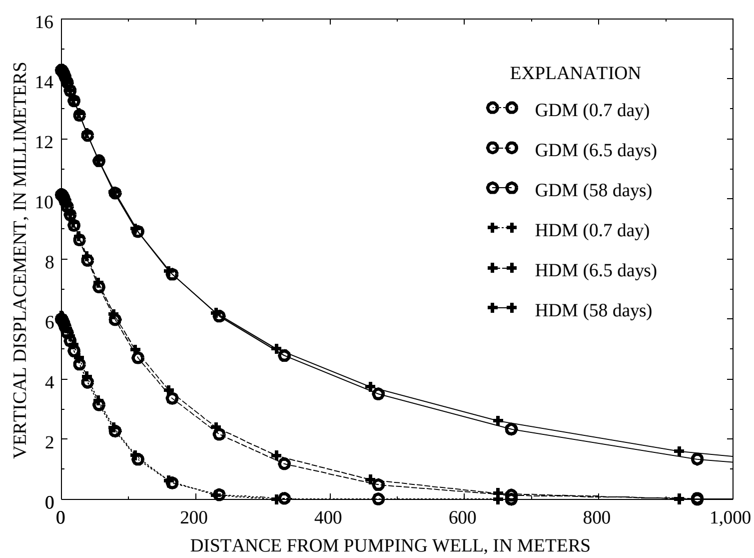

Figure 2. Simulated vertical displacement for three time periods using the granular displacement model (GDM) and the Hsieh displacement model (HDM).

California—1. Constant parameters: American Geophysical Union, Water Resources Research, v. 11, no. 3, p. 465–478.

___1976, One-dimensional simulation of aquifer system compaction near Pixley, California—2. Stress-dependent parameters: American Geophysical Union, Water Resources Research, v. 12, no. 3, p. 375–391.

___1994a, Hydraulic forces that play a role in generating fissures at depth: Bulletin of the Association of Engineering Geologists, v. 31, no. 3, p. 293–303.

___1994b, Horizontal aquifer movement in a Theis-Thiem confined system: American Geophysical Union, Water Resources Research, v. 30, no. 4, p. 953–964.

Holzer, T.L., 1984, Ground failure induced by ground-water withdrawal from unconsolidated sediment: Geological Society of America, Reviews in Engineering Geology, v. 6, p. 67–105.

Leake, S.A., 1990, Interbed storage changes and compaction in models of regional ground-water flow: American Geophysical Union, Water Resources Research, v. 26, no. 9, p. 1939–1950.

___1991, Simulation of vertical compaction in models of regional ground-water flow, *in* Johnson, I.A., ed., Land Subsidence: International Association of Hydrological Sciences Publication 200, Proceedings of the Fourth International Symposium on Land Subsidence, p. 565–574.

Leake, S.A., and Prudic, D.E., 1991, Documentation of a computer program to simulate aquifer-system compaction using the modular finite-difference ground-water flow model: U.S. Geological Survey Techniques of Water-Resources Investigations, book 6, chap. A2, 68 p.

McDonald, M.G., and Harbaugh, A.W., 1988, A modular three-dimensional finite-difference ground-water flow model: U.S. Geological Survey Techniques of Water-Resources Investigations, book 6, chap. A1, 586 p.

Meyer, W.R., and Carr, J.E., 1979, A digital model for simulation of ground-water hydrology in the Houston area, Texas: Texas Department of Water Resources Report LP–103, 27 p.

Poland, J.F., and Davis, G.H., 1969, Land subsidence due to withdrawals of fluids, *in* Varnes, D.J., and Kiersch, George, eds., Reviews in Engineering Geology—Volume 2: Boulder, Colorado, Geological Society of America, p. 187–269.

Rice, J.R., and Cleary, M.P., 1976, Some basic stress diffusion solutions for fluid-saturated elastic porous media with compressible constituents: Review of Geophysics and Space Physics, v. 14, p. 227–241.

Trescott, P.C., 1975, Documentation of finite-difference model for simulation of three-dimensional ground-water flow: U.S. Geological Survey Open-File Report 75–438, 32 p.

Wolff, R.G., 1970, Relationship between horizontal strain near a well and reverse water-level fluctuation: American Geophysical Union, Water Resources Research, v. 6, no. 6, p. 1721–1728.