Poroelasticity Simulation of Ground-Water Flow and Subsurface Deformation

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INTRODUCTION

Withdrawal of ground water causes horizontal and vertical displacements in the subsurface. If the subsurface material is assumed to be linearly elastic, then poroelasticity theory, originally developed by Biot (1941), can be used to analyze the coupled interaction between ground-water flow and subsurface (matrix) deformation. In this study, a finite-element model is developed to solve the axisymmetric form of the poroelasticity equations. The model is used to analyze deformation-induced changes in hydraulic head, a phenomenon also known as the Noordbergum effect or reverse water-level fluctuation.

POROELASTICITY MODEL

In poroelasticity theory, fluid flow is described by Darcy's Law and mass conservation, and matrix deformation is described by Biot's constitutive relations and stress equilibrium. Strains are assumed to be small. In this study, it is further assumed that (a) the subsurface is in an initial state of hydraulic and mechanical equilibrium, (b) gravitational body force remains constant, (c) the matrix grains are incompressible, and (d) the fluid is compressible. Under the above assumptions, the equations of poroelasticity are (Verruijt, 1969, p. 342):

$$\kappa\nabla^2 h = \frac{\partial}{\partial t}(\nabla \bullet \underline{u}) + \frac{\rho_f g n}{K_f}\frac{\partial h}{\partial t} , \qquad (1)$$

and

$$G\nabla^2 \underline{u} + \frac{G}{1-2\nu}\nabla(\nabla \bullet \underline{u}) - \rho_f g \nabla h = 0 , \qquad (2)$$

where h is change in hydraulic head from the initial head, $\underline{u}$ is displacement vector of the skeletal matrix, t is time, κ is hydraulic conductivity, n is porosity, $\rho_f g$ and K_f are the specific weight and bulk modulus of the fluid, respectively, and G and ν are the shear modulus and drained Poisson's ratio of the skeletal matrix, respectively. A numerical model has been developed to solve the axisymmetric form of equations (1) and (2) following the finite-element method of Smith and Griffiths (1988, chap. 9). In this formulation, material properties are assumed uniform within each element, but properties may vary from one element to the next. The model, therefore, may be applied to solve problems with a nonuniform distribution of properties.

SIMULATION OF DEFORMATION-INDUCED CHANGES IN HYDRAULIC HEAD

The poroelasticity model is applied to simulate deformation-induced changes in hydraulic head that often are observed in aquitards when water is pumped from an adjacent aquifer. A hypothetical setting consists of a 100-meter thick, laterally extensive aquifer that is confined above and below by aquitards (fig. 1). The upper aquitard is 100 m thick, and the lower aquitard extends to a great depth. The water table coincides with the land surface, which is the top of the upper aquitard. A well is screened over the entire thickness of the aquifer, and for simplicity, the well is assumed to be cased throughout both aquitards. Ground water is pumped at a constant rate of 5×10^{-2} m^3/s. The hydraulic and mechanical properties used in the simulation are shown in table 1. The aquifer

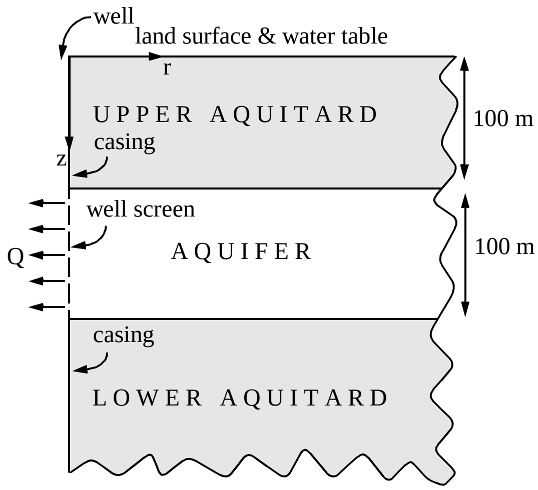

Figure 1. Hypothetical setting of an aquifer confined above and below by aquitards.

properties are characteristic of an unconsolidated, sandy formation. The two aquitards have identical properties that are characteristic of unconsolidated silty deposits.

In the numerical model, the aquifer and aquitards extend laterally from the well radius of 0.1 m to an outer boundary 10 km away. The outer boundary is impervious, and there is no change in stresses. The lower aquitard extends vertically to a bottom boundary 10 km below land surface. The bottom boundary is impervious, and there is no displacement. These two remote boundaries are sufficiently distant from the well screen so that slight variations in boundary conditions would not affect the solution in the region affected by pumping. The top boundary (land surface) is free of boundary traction, and drawdown of the water table is assumed negligible. At the well, there is no radial displacement and no change in the vertical component of boundary traction. This boundary condition allows the matrix along the well screen and (or) casing to move vertically but not horizontally. A uniform flux of water is withdrawn from the entire thickness of the aquifer so that no flow crosses the casing in the aquitards. The 10-kilometer by 10-kilometer model domain is discretized into a 40-column by 100-row mesh of rectangular elements with variable sizes. To prevent numerical oscillation, elements as thin as 0.1 m are used along aquifer-aquitard interfaces. The first time step is 15 seconds, and this time-step size is successively increased by 1.2 times until a total simulation time of 50 days is reached.

A 300- by 300-meter vertical section of aquifer and aquitards in their initial, undeformed states (before pumping) is shown in figure 2*A*. The well is on the left, land surface is at the top, and only the upper 100 m of the lower aquitard is shown. Horizontal and vertical grid lines, spaced 20 m apart, are superimposed on the section. By moving with the skeletal matrix, these grid lines illustrate the deformation of the aquifer and aquitards during pumping. Note that the grids shown in figure 2 are not the finite-element mesh used in the numerical model.

The simulated deformation of the aquifer and aquitards after 10 minutes of pumping is shown in figure 2*B*. For the sake of illustration, displacements are exaggerated 40,000 times so that, for example, a displacement of 40 m in figure 2*B* would correspond to an actual displacement of 1 mm. Contraction of the aquifer is evident. Horizontal contraction near the well is seen by the deflection of vertical grid lines towards the screen. A point initially at A in figure 2*A* has moved about 0.1 mm in the horizontal direction to A' in figure 2*B*. Vertical contraction can be seen by a

Table 1. Values of physical properties used in simulation

Physical property	Value in aquifer	Value in aquitards
Hydraulic conductivity, κ, in meters per second	1×10^{-4}	1×10^{-7}
Shear modulus, *G*, in Newton per square meter	3×10^{8}	3×10^{7}
Drained Poisson's ratio, ν (dimensionless)	.25	.25
Porosity, *n* (dimensionless)...	.30	.40
Fluid bulk modulus, K_f, in Newton per square meter	2.3×10^{9}	2.3×10^{9}

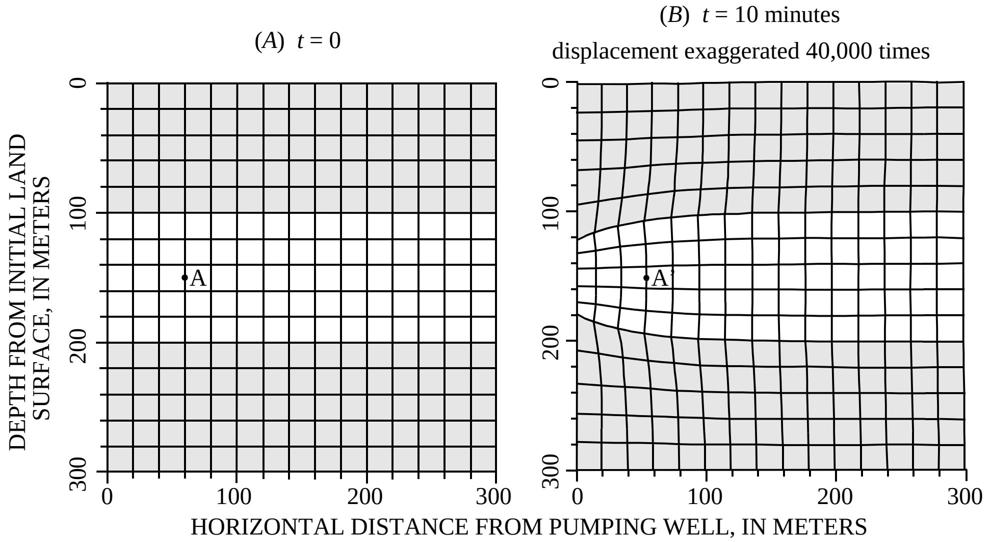

Figure 2. Deformation of a 300- by 300-meter vertical section of aquifer and aquitards with grid lines superimposed. Unshaded area indicates aquifer. Shaded area indicates aquitard. *t* is time since pumping began.

decrease of aquifer thickness. Immediately adjacent to the well, the decrease of thickness is about 1 mm, and the aquitards also have deformed. Near the well, the aquitards have contracted in the horizontal direction and extended in the vertical direction, and shear distortion increases toward the aquifer-aquitard interface.

The horizontal contraction and vertical extension in the aquitards cause local changes of pore volume. In some parts of the aquitard, pore volume increases, and in other parts, pore volume decreases. Understanding the relation between pore-volume change and head change is the key to understanding deformation-induced effects. During early time after the start of pumping, there is essentially no fluid flow in the aquitards. Under this condition, change in hydraulic head is inversely proportional to change in pore volume. At any point in the aquitard, if deformation results in a net increase in pore volume, hydraulic head drops. Conversely, if deformation results in a net decrease in pore volume, hydraulic head rises.

Progressive changes occur in hydraulic head in the 300- by 300-meter vertical section of aquifer and aquitards (fig. 3*A*–*F*). After 10 minutes of pumping (fig. 3*A*), two zones of induced head drop have developed—one above and one below the well screen. The head drops in both zones generally are between 0 and 1 cm. (The contour line for 1 cm of head drop lies close to the aquifer-aquitard interface). These two zones are regions of pore-volume increase in the aquitards. Two separate zones of induced head rise emerge in the upper aquitard. The shallower zone is just below land surface and near the well. The deeper zone occurs at the base of the aquitard and about 50 m radially outward from the well. The zones are the regions of pore-volume reduction in the aquitard.

As pumping continues, the aquifer contracts further and induces greater head changes in the aquitards. The maximum head rise exceeds 3 cm after 1 hour of pumping (fig. 3*B*), and 6 cm after 6 hours (fig. 3*C*). Head rise is greater in the upper aquitard than in the lower aquitard. The asymmetry is due to the presence of land surface where the absence of traction allows more deformation of the near-surface material. Of the two zones of head rise in the upper aquitard, the shallower one (just below land surface) has greatly expanded, and the deeper one has almost merged with the expanding contours. Of the two zones of head rise, the one above the well screen has almost disappeared, and the one below the well screen has remained about the same. These results show that, in the present case, head rise is the dominant effect induced by

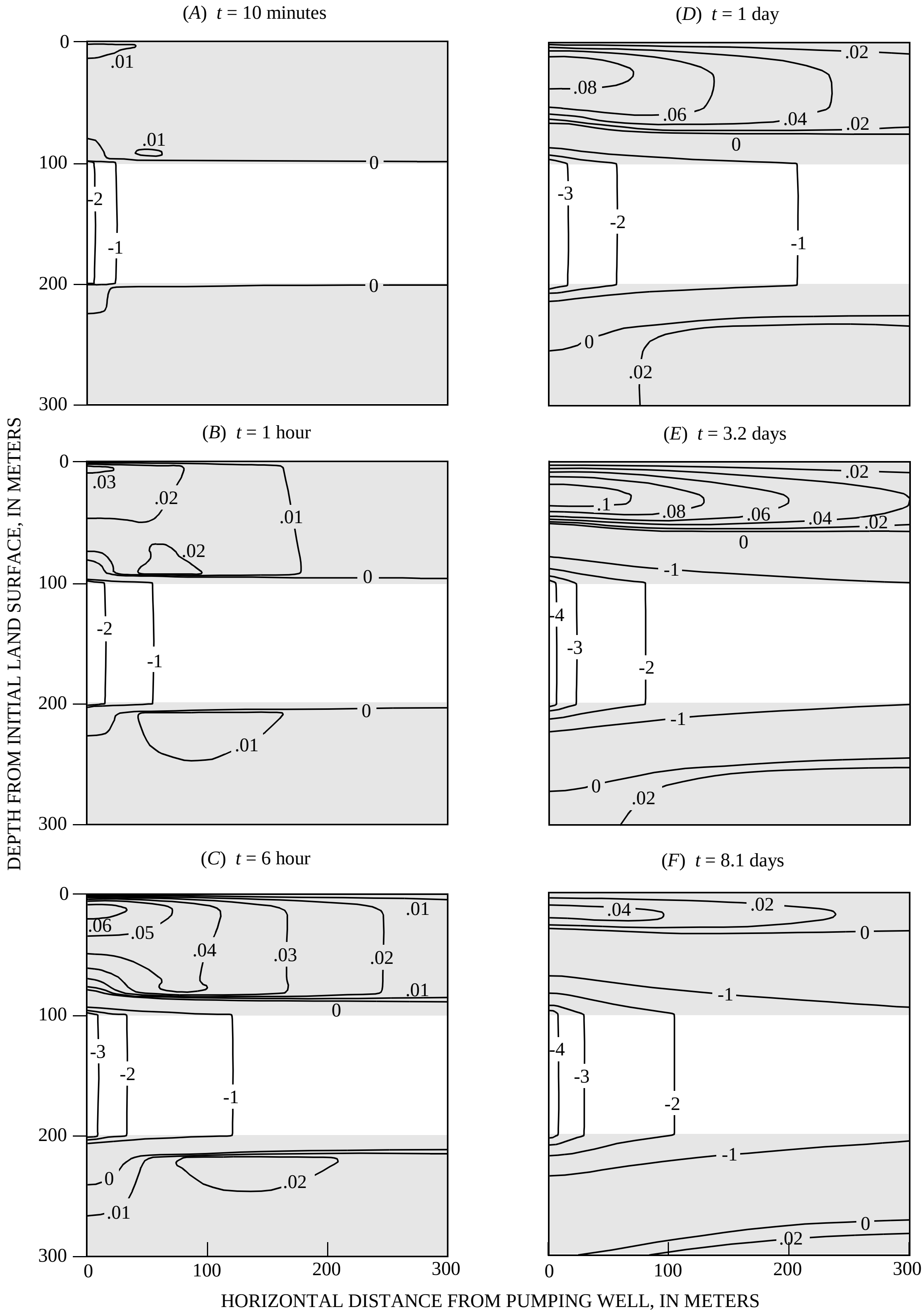

Figure 3. Changes in hydraulic head, in meters, in 300- by 300-meter vertical section of aquifer and aquitards. Unshaded area indicates aquifer. Shaded area indicates aquitard. Positive numbers indicate head rise, in meters. Negative numbers indicate head drop, in meters, Contour interval is irregular. *t* is time since pumping began.

pumping. Head drop is confined to a relatively small region above and below the well screen.

The deformation-induced head rise is dissipated over time by: (1) fluid flow from regions of higher head to regions of lower head (including the water table) and (2) propagation of head drop from the aquifer into the aquitards (fig. 3*D*, *E*, and *F*). After 1 day of pumping, head drop has propagated into the lower quarter of the upper aquitard. Above this zone of head drop, hydraulic head is still increasing (exceeding 8 cm), but flow to the water table has displaced the maximum to a lower position than before. After 3.2 days, the maximum head rise exceeds 10 cm; however, the region of head rise in the upper aquitard now encompasses only its upper half. By 8.1 days, the maximum head rise has decreased to about 4 cm, indicating significant dissipation of deformation-induced effects. After 14 days (not shown), head drop has propagated throughout the upper aquitard, and deformation-induced effects are no longer observable.

CONCLUSIONS

A simulation using a poroelasticity model with typical aquifer and aquitard properties shows that three-dimensional deformation caused by ground-water withdrawal from a confined or semiconfined aquifer can induce hydraulic-head changes in adjacent aquitards. The deformation-induced head changes range from about 1 cm of head drop to about 10 cm of head rise. These results are consistent with reported field observations. The simulation suggests that in a thick aquitard, induced head rise could persist for many days after the start of pumping but is eventually dissipated by (1) fluid flow from regions of higher head to regions of lower head and (2) propagation of drawdown from the pumped aquifer into the aquitard. The poroelasticity model developed for this analysis is useful in understanding the hydraulic response to ground-water withdrawal in layered aquifer-aquitard systems.

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