Simulation of Deformation of Sediments from Decline of Ground-Water Levels in an Aquifer Underlain by a Bedrock Step

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INTRODUCTION

Land subsidence occurs in many areas where ground-water pumping lowers water levels within compressible aquifer systems. In the southwestern United States, aquifer-system compaction and land subsidence have resulted in earth fissures—particularly along the margins of aquifers in alluvial basins that have extensive ground-water pumpage. One explanation for the development of fissures is horizontal extensional strain that could develop over buried bedrock highs or steps (Jachens and Holzer, 1982; Carpenter, 1993). Commonly applied methods of evaluating aquifer-system compaction (Leake and Prudic, 1991) are applications of the Terzaghi theory, in which all compaction is assumed to be vertical. Neglecting the horizontal components of deformation does not allow for complete analysis of problems of earth fissuring. Furthermore, the one-dimensional approach raises questions regarding the validity of computed distributions of head and vertical deformation in systems where horizontal deformation is significant. The more rigorous Biot (1941) theory of deformation accounts for horizontal as well as vertical components of elastic deformation. The purpose of this analysis is to evaluate relative importance of horizontal and vertical components of deformation in an aquifer underlain by a bedrock step.

APPROACH

Deformation around a bedrock step was simulated in a vertical plane for an idealized aquifer system (fig. 1). The right side of the plane is analogous to an edge of an alluvial basin, and the left side is analogous to an interior part of a basin where ground-water pumping causes water levels to decline. The aquifer system includes an upper unconfined compressible aquifer, a middle compressible confining unit, and a lower compressible confined aquifer.

Ground-water flow and deformation in the hypothetical system were simulated using two different methods. The first method used the MODFLOW finite-difference ground-water model program (McDonald and Harbaugh, 1988) to simulate ground-water flow in the vertical plane. Vertical compaction, Δb, is computed using the relation

$$\Delta b = -S_{sk} b \Delta h ,$$

where S_{sk} is the skeletal component of specific storage, b is thickness of compacting sediments, and Δh is change in head computed by MODFLOW. The relation is consistent with the Terzaghi theory (Leake and Prudic, 1991, equation 5). Compaction is computed for each cell, and vertical displacement is computed as half of the compaction for a cell plus the sum of compaction values for the cells below. The second method is a finite-element solution of flow and deformation in two dimensions. Fluid flow is described by Darcy's Law and mass conservation, and matrix deformation is described by Biot's constitutive relations and stress equilibrium. For more details on this method, see paper entitled "Poroelasticity Simulation of Ground-Water Flow and Subsurface Deformation" on page 5 in this report. In this paper, the two methods are referred to as the Terzaghi method and the Biot method, respectively.

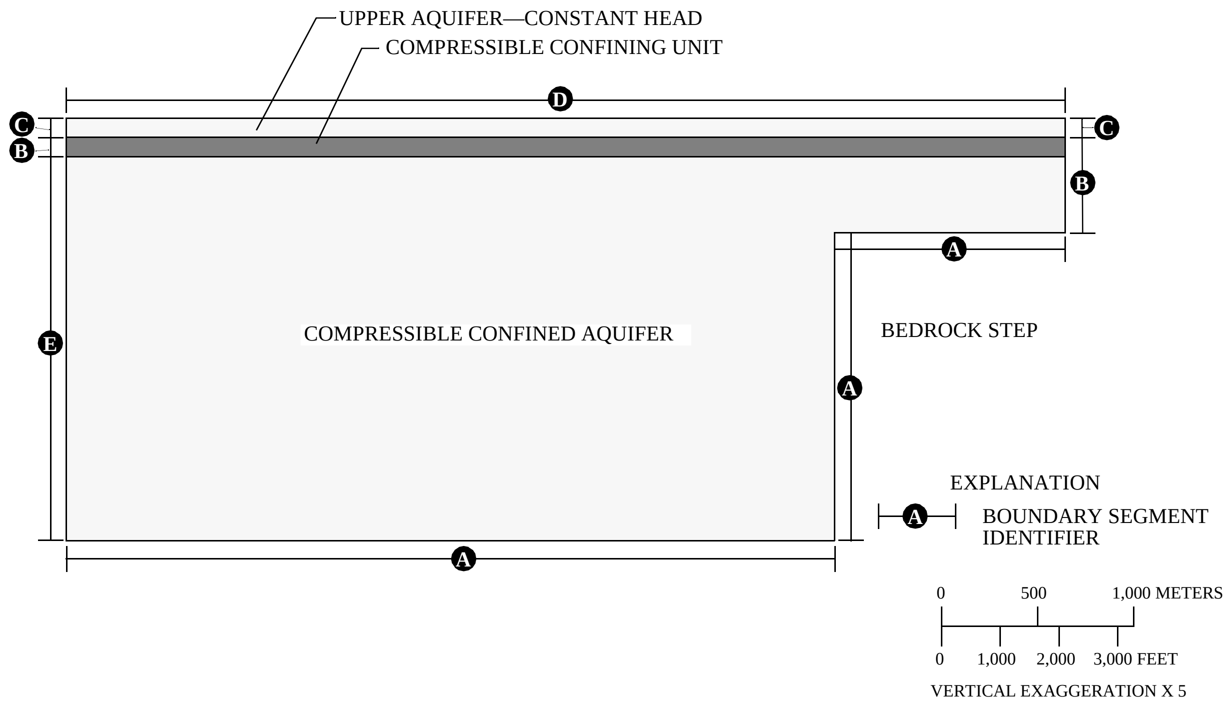

Figure 1. Hydrologic system used in simulations of flow and deformation.

For simplicity in simulating flow and deformation in vertical sections, the head in the upper aquifer was specified to remain constant for the entire period of simulation; therefore, no compaction occurs in that aquifer. The hydraulic boundary conditions (table 1) allow for a specified decline in head in the lower aquifer along the left side of the model. Along boundary-segment E (fig. 1), head is specified to linearly decline by 60 m over a 10-year period (fig. 2).

The Terzaghi method as applied for this analysis does not compute displacements along the edges of the model, but allows for vertical displacement at the centers of all model cells. The mechanical boundary conditions for the Biot method (table 1) allow for vertical displacement everywhere except along the lower boundary including the step. Horizontal displacement is allowed along the top boundary.

In the finite-difference and finite-element models, the hydrologic system is represented using 26 horizontal rows and 65 vertical columns of cells and elements. Rows are 20 m thick in the aquifers and 4 m thick in the confining unit, and each column is 80 m wide. The hydraulic and physical properties used in the simulations are given in table 2. The values are within ranges expected in alluvial basins in the southwestern United States (Hanson, 1989). Simulations with the Terzaghi method used a single skeletal specific-storage

Table 1. Boundary conditions used in simulations of flow and deformation

Boundary-segment identifier	Hydraulic boundary condition	Mechanical boundary condition used in Biot method
A	No flow	No horizontal or vertical displacement
B	No flow	No horizontal displacement
C	Specified head (constant)	No horizontal displacement
D	Specified head (constant)	Free surface
E	Specified head (declining)	No horizontal displacement

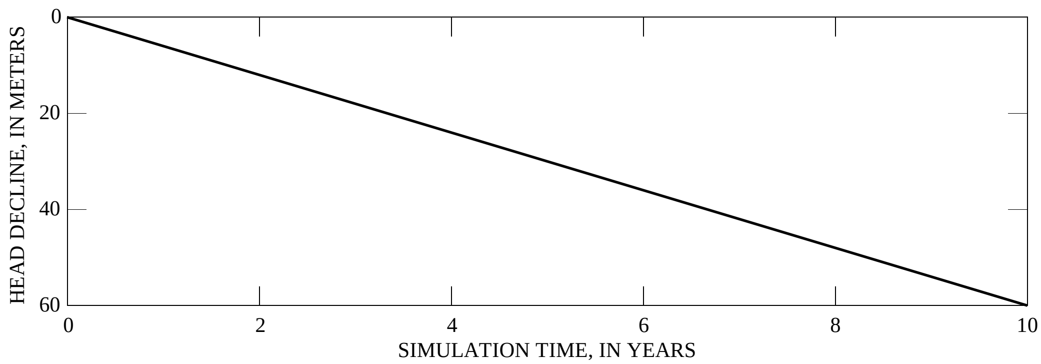

Figure 2. Specified decline in head along boundary-segment E.

Table 2. Hydraulic and physical properties used in simulations of flow and deformation

Hydrologic unit	Hydraulic conductivity (meter day^{-1})	Skeletal specific storage (meter^{-1})	Young's modulus (Newton meter^{-1})	Poisson's ratio
Upper aquifer	25	1×10^{-5}	8×10^{8}	0.25
Middle confining unit	.01	1×10^{-4}	8×10^{7}	.25
Lower aquifer...........................	25	1×10^{-5}	8×10^{8}	.25

value rather than elastic and inelastic specific-storage values. The values of 1×10^{-5} m^{-1} for the aquifers and 1×10^{-4} m^{-1} for the confining unit are in the range of typical values for the skeletal component of inelastic specific storage. Using these values, this analysis is valid only for the case in which pore pressures do not increase (fig. 2). The Biot method does not use specific storage but rather uses Young's modulus and Poisson's ratio. The values given in table 2 are consistent with the specific-storage values used in the Terzaghi method. Although the Biot method is applicable to an elastic medium, the application here with properties corresponding to typical inelastic specific-storage values is appropriate as long as the Young's modulus and Poisson's ratio can be assumed to remain constant.

RESULTS

A comparison of results of the two methods can be used to evaluate the effects of ignoring horizontal deformation on computed head change. The results for the 10-year simulations using the Terzaghi and Biot methods are the same except at the extreme edge of the basin (fig. 3*A*); therefore, the assumptions inherent in the Terzaghi method have little effect on the computation of head change. Similarly, computed vertical displacement is almost identical over most of the area (fig. 3*B*). Within a distance of about one aquifer thickness on each side of the bedrock step, the distributions differ. The distribution computed by the Terzaghi method includes a discontinuity over the bedrock step. The displacement in the upper cell in the thin part of the aquifer adjacent to the step is less than 0.08 m, and the displacement in the upper cell in the thick part of the aquifer adjacent to the step is more than 0.18 m. The displacement in the upper cell in the thin part of the aquifer computed by the Biot method also is less than 0.08 m; however, the displacement computed by the Biot method in the upper cell in the thick part of the aquifer is only 0.10 m. Furthermore, the Biot method results in a smooth continuous distribution of displacement across the step.

The results of the Biot method can be useful for analysis of conditions leading to earth fissuring. The horizontal strain at land surface computed by the method (fig. 4) shows a tensional strain of more than 1×10^{-4} on the upside of the step. Although strain at failure is not well known, Jachens and

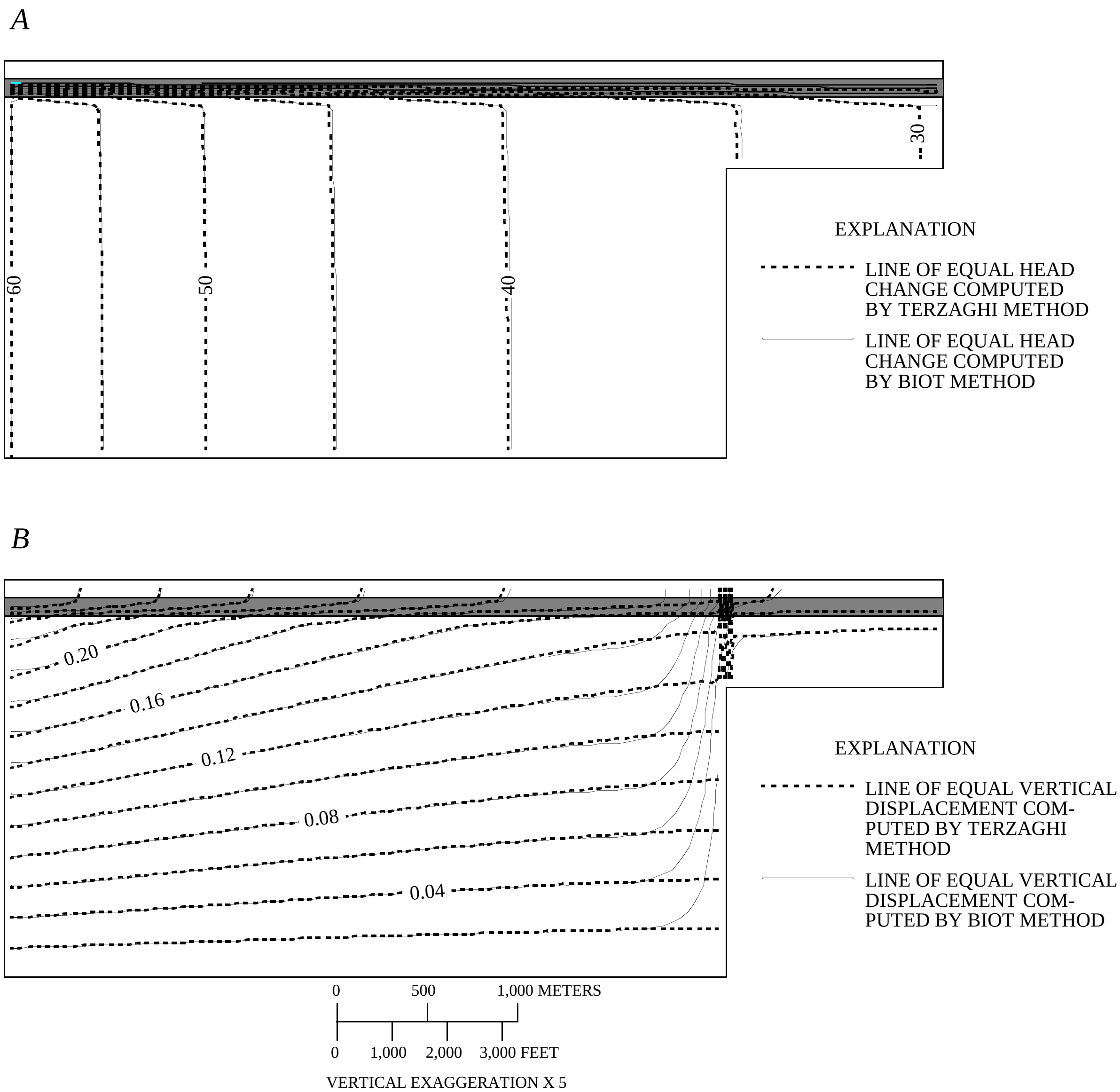

Figure 3. Results of simulations of flow and deformation using the Terzaghi and Biot methods. *A*, Computed head change, in meters. *B*, Computed vertical displacement, in meters.

Holzer (1982) calculate values in the range of 2×10^{-4} to 2×10^{-3}.

CONCLUSIONS

Head changes computed by the Terzaghi and Biot methods were almost identical throughout the simulated flow system. Vertical displacements computed by the two methods were almost the same except near the bedrock step. For the conditions simulated, the simpler Terzaghi method adequately describes deformation on a regional scale. For analysis of smaller-scale deformation around the bedrock step, the more rigorous Biot method is needed to adequately simulate vertical and horizontal deformation. The deformation computed by the Biot method can be used to calculate potential horizontal strains that lead to development of earth fissures.

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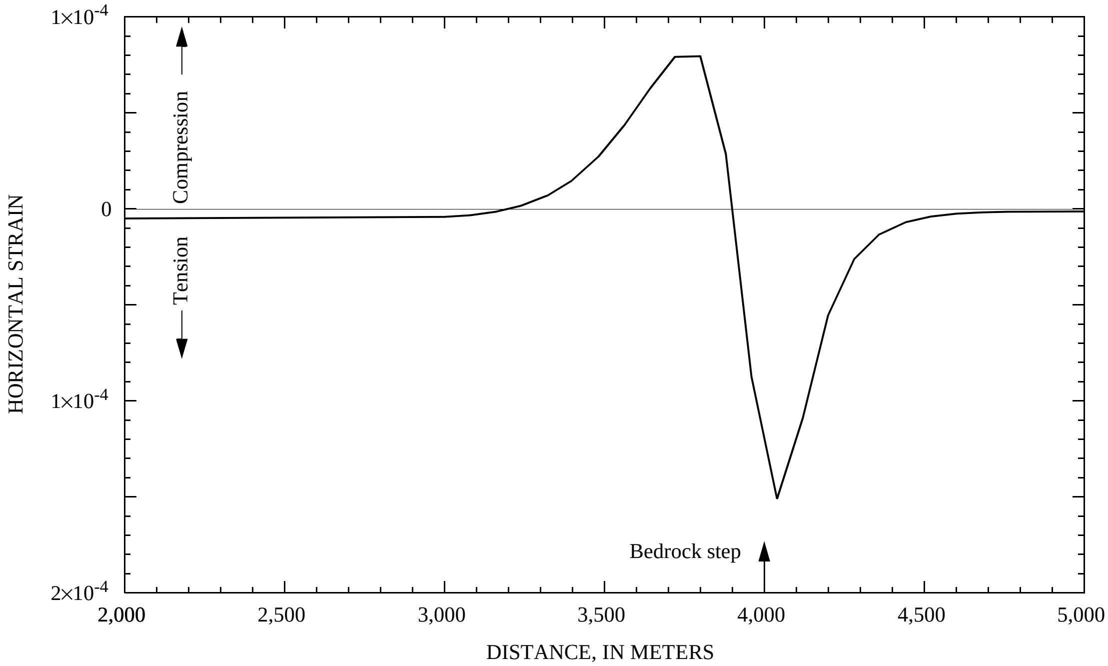

Figure 4. Horizontal strain at land surface at the end of the 10-year simulation computed by the Biot method.

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