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Comment on the Treatment of Residual Water Content in "A Consistent Set of Parametric Models for the Two-Phase Flow of Immiscible Fluids in the Subsurface" by L. Luckner et al.

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Luckner et al. [1989] (hereinafter LVN) present a clear summary and generalization of popular formulations used for convenient representation of porous media fluid flow characteristics, including water content (θ) related to suction (h) and hydraulic conductivity (K) related to θ or h. One essential but problematic element in the LVN models is the concept of residual water content (θ_r; in LVN, $\theta_{w,r}$). Most studies using θ_r determine its value as a fitted parameter and make the assumption that liquid flow processes are negligible at θ values less than θ_r. While the LVN paper contributes a valuable discussion of the nature of θ_r, it leaves several problems unresolved, including fundamental difficulties in associating a definite physical condition with θ_r, practical inadequacies of the models at low θ values, and difficulties in designating a main wetting curve.

The LVN paper (p. 2188) defines θ_r as the value of θ at which films of wetting liquid coating the solid particles are reduced to the point where "all or parts of the connecting films become so thin, and hence so strongly adsorbed onto the solid phase, that the wetting fluid loses its capability to respond to hydraulic gradients." It is highly desirable to have such a physically based definition, but this particular definition is not well supported by observation. The possibility that liquid flow might cease at a nonzero θ value has been investigated, but so far there is no conclusive experimental evidence that such a condition exists. Even if it does, there is no evidence that commonly cited θ_r values (sometimes as large as 0.2 or more [e.g., *van Genuchten*, 1980]) represent the θ at which liquid flow ceases. On the contrary, an experimental investigation with a sandy soil (porosity 0.335) at $\theta = 0.088$ has shown not only that liquid flow occurs under conditions near θ_r (determined by curve fitting to be 0.076), but that it closely obeys Darcy's law [*Nimmo et al.*, 1987].

If the described no-liquid-flow condition does exist, it would be no surprise if θ_r determined from it differed substantially from θ_r determined from $\theta(h)$ curve fits. The condition described by LVN is based on dynamic phenomena, so $\theta(h)$ curves, being based on static liquid retention properties, have no necessary fundamental link with it.

Apart from definitional difficulties, it is unrealistic to represent a $\theta(h)$ curve as never becoming less than θ_r. In reality, as h continues increasing, θ continues decreasing until it is 0. Vapor transport eventually becomes dominant over liquid transport (see, e.g., *Rose* [1963]), but this fact does not require the curve to remain above any particular θ value. Figure 1 illustrates this fact with data of *Schofield* [1935]. These data, for a loam, were obtained using four different measurement techniques to cover the complete moisture range. The data closely approach water content 0 (defined by standard ovendry conditions) at a suction of about 7×10^6 cm water. The LVN paper does effectively acknowledge that θ eventually becomes 0 as h increases, in discussing vapor transport processes in dry soils, and also the "primary wetting curve" (PWC) in LVN Figure 2.

Although the basic nature of θ_r is problematical, it is generally necessary to include it as a fitting parameter in models that employ *van Genuchten*'s [1980] empirical $\theta(h)$ formula (generalized in LVN equation (6)). While this formula offers computational advantages and often represents hydraulic characteristics reasonably well at high θ, it works poorly near the value chosen as θ_r and not at all below it. The curves in Figure 1 illustrate some of the problems that can arise. Curve A ($\theta_r = 0.018$) was determined by optimizing the parameters of this formula in a fit to all the data. It clearly fits better in the wet than in the dry range. The dry range discrepancy might be acceptable for some purposes, but the systematic nature of the deviation (undervaluing θ in the middle range and overvaluing it in the extreme dry range) is characteristic of the empirical formula and gives a serious undervaluation of specific water capacity. Curves B and C were fit only to the points with $h < 2000$ cm water, curve B with an optimized θ_r (0.142) and C with θ_r fixed at 0. They show that for the usual situation in which only wet range measurements are available, the fitted curve is more realistic

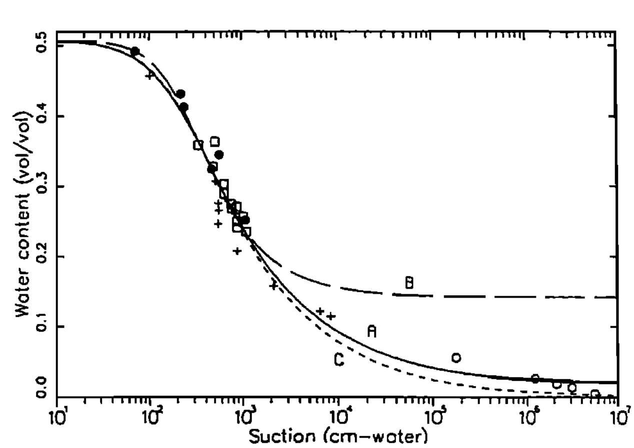

Fig. 1. Water content versus suction data of *Schofield* [1935], with fitted curves of the LVN model. Four different experimental techniques were used to cover a five-order-of-magnitude range of h. Different symbols distinguish the techniques: solid circles for direct suction, open squares for centrifuge, pluses for freezing point, and open circles for vapor pressure.

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when θ_r is fixed at 0. Although C is a much better fit than B, it has defects similar to those of A while, as expected, representing the dry range data less well.

Another low-θ problem, apparent in LVN Figure 2, is that the PWC is not as closely analogous to the primary drying curve (PDC) as implied by LVN. The wetting and nonwetting phases have fundamentally different behavior at the extremes of θ, near 0 and near saturation. Considering the nonwetting phase, the air-trapping phenomenon leads to a clear distinction between the main drying curve (MDC) and the PDC. The absence or presence of trapped air at $h = 0$ distinguishes primary and nonprimary drying curves. Even if an analogous condition exists for the wetting phase, it cannot be similarly associated with a specific h value. Without such a definitive starting point, a wetting curve can begin from any θ between 0 and ϕ, so the choice of θ_r for the main wetting curve (MWC) is arbitrary.

In LVN Figure 2 the difference between the curves designated PWC and MWC may be significantly exaggerated. The capillary mechanism responsible for the greatest degree of soil moisture hysteresis operates only above a certain θ value, and below that value even adsorptive mechanisms of hysteresis eventually become negligible [*Brunauer*, 1945, p. 394]. Thus it is likely that below some θ value (perhaps near the values commonly chosen as θ_r), there is so little hysteresis that all the curves (drying and wetting, primary and nonprimary) are essentially identical. Then PWC would deviate from MWC only at greater θ values and perhaps by only a small amount. The LVN models do not represent such differences as small unless θ_r is essentially 0. A better interpretation of this graph would be to drop the concept of a PWC, designate the curve labeled PWC as the actual MWC, and consider the curve labeled MWC as a scanning curve. As the LVN paper points out, it is unfortunate that very few wetting curves starting from $\theta = 0$ have been measured. If it is true, however, that these curves are much closer together than LVN Figure 2 indicates, then the experimental convenience of using a scanning curve starting from a low but nonzero θ as the MWC would cause little error.

These conceptual and practical shortcomings related to θ_r limit use of the LVN models to systems with θ well above what would be designated as θ_r. Although this has probably not been a problem for most porous media topics that have been of interest in the past, more and more research, for example in arid region hydrology, requires adequate representation of characteristics for small as well as large θ values. When practical models as presented by LVN seem to be inadequate for these applications, the deficiency is probably in the models themselves rather than the underlying unsaturated flow theory; the basic theory may well be quite sound near and below the values given as θ_r. Besides the low-θ Darcy's law test mentioned earlier, a more recent study [*Nimmo*, 1990] of transient liquid flow in a sandy soil in the 0.08–0.12 θ range has shown fair support for the validity of Richards' equation. These results suggest that the same basic physical processes that are important in wet soil still are of chief importance well into the dry range. They also underline the desirability of practical models that are valid over the whole range of conditions for which the underlying theory is valid, as well as for which there are applications of active interest.

The criticism I have outlined here could be directed to various other popular models and formulations that are inadequate at low θ values. In fact the LVN presentation is superior because it discusses possible modes of vapor transport and it raises the issue of the physical interpretation of θ_r. It is not surprising that different points of view arise concerning the behavior of water at low θ because little relevant experimental evidence is available. The LVN and similar models have limitations that derive from a flawed θ_r concept. To develop improved models that are not so limited, if the θ_r concept is retained, it will be necessary to develop for it a consistent theoretical basis which can be supported by experimental observations.

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