WATER RESOURCES RESEARCH, VOL. 27, NO. 6, PAGES 1263–1269, JUNE 1991

Centrifugal Techniques for Measuring Saturated Hydraulic Conductivity

JOHN R. NIMMO AND KAREN A. MELLO

Water Resources Division, U.S. Geological Survey, Menlo Park, California

Centrifugal force is an alternative to large pressure gradients for the measurement of low values of saturated hydraulic conductivity (K_{sat}). With a head of water above a porous medium in a centrifuge bucket, both constant-head and falling-head measurements are practical at forces up to at least 1800 times normal gravity. Darcy's law applied to the known centrifugal potential leads to simple formulas for K_{sat} that are analogous to those used in the standard gravity-driven constant- and falling-head methods. Both centrifugal methods were tested on several fine-textured samples of soil and ceramic with K_{sat} between about 10^{-10} and 10^{-9} m/s. The results were compared to falling-head gravity measurements. The comparison shows most measurements agreeing to within 20% for a given sample, much of the variation probably resulting from run-to-run changes in sample structure. The falling-head centrifuge method proved to be especially simple in design and operation and was more accurate than the constant-head method. With modified apparatus, K_{sat} measurements less than 10^{-10} m/s should be attainable.

INTRODUCTION

The direct measurement of saturated hydraulic conductivity (K_{sat}) becomes difficult at values less than the 10^{-3} to 10^{-9} m/s range commonly observed in agricultural soils and subsurface reservoirs. Direct measurements require the artificial generation of flow under conditions in which all relevant quantities can be measured, so for low K_{sat} it is necessary to measure very small flow volumes or to drive the flow with great pressure gradients, or both.

An alternative to high pressure is centrifugal force. A body force such as that generated in a centrifuge is different from pressure-generated forces in several respects, leading to various advantages and disadvantages. Some clear advantages are that (1) the centrifugal force is easily adjustable and reproducible merely by setting the centrifuge speed, (2) the force is confined to apparatus within the centrifuge buckets, (3) since it goes as the square of rotational speed, the force can accommodate a huge range of K values with the same apparatus and procedures, (4) the flow apparatus can be extremely simple, requiring no regulators, valves, transducers, or complicated plumbing, and (5) since it acts on all components of the medium, centrifugal force can simulate overburden pressures as it generates driving force.

A common application of low-K_{sat} measurements is in the study of flow in low-permeability geological materials [*Neuzil*, 1986]. A second, increasingly important application, is in assessing the effectiveness of artificial barriers to underground flow, as for hazardous waste containment [*Mundell and Bailey*, 1984]. Even for less demanding applications, when higher K_{sat} values must be measured, centrifugal techniques using low rotational speeds may be desirable for rapidity and simplicity of operation.

The constant-head and falling-head techniques with gravity driving the flow [*Klute and Dirksen*, 1986] are in routine use, especially for soil science applications. These methods commonly measure K_{sat} values down to about 10^{-10} m/s. For lower K_{sat} values, widely used techniques include the use of large, steady pressure gradients [e.g., *Ohle*, 1951]; falling heads with large pressure [e.g., *Terzaghi*, 1925]; and observation of transient pressure and volume changes, calibrated by knowing the compressive properties of experimental devices [e.g., *Bianchi and Haskell*, 1963] and sometimes also of the fluid and medium [e.g., *Brace et al.*, 1968].

Previous measurements of hydraulic conductivity with a centrifuge have dealt with unsaturated flow [*Alemi et al.*, 1976; *Nimmo et al.*, 1987]. The patent of the steady state centrifuge method (SSCM) [*Rubin et al.*, 1987] mentions the extension of this technique to saturated media. As concerns the necessary apparatus and operation, this extension is actually a simplification. D. P. Hammermeister (unpublished data, 1980) developed and used a variation of SSCM to measure K_{sat} of ceramic plates.

Given that the use of centrifugal force may permit easier, faster measurements of low K_{sat} values, the main objective of the present study was to apply easily implemented centrifugal techniques to different porous media, showing that both steady flow and falling-head methods for K_{sat} can operate in a centrifugal field. Then the relative advantages of both techniques can be compared to each other and to noncentrifugal techniques.

DESCRIPTION OF METHODS

Constant Head

The constant-head centrifuge method (abbreviated CC) uses the experimental configuration diagrammed in Figure 1. Centrifugal force drives water through the sample, starting from a reservoir above the sample. The upper boundary condition is atmospheric pressure at the air-water interface, positioned at radius r_{wa} from the axis of rotation. The lower boundary condition in general is similarly determined by an air-water interface at r_0. It is often convenient not to maintain a free water surface at a position between r_t and r_b but instead to permit water to leave the sample at atmospheric pressure, in effect letting r_0 equal r_b.

The CC method requires a regulating mechanism that maintains the air-water interface at a fixed position. Then the flow becomes steady with a rate determined according to Darcy's law:

Paper number 91WR00367.

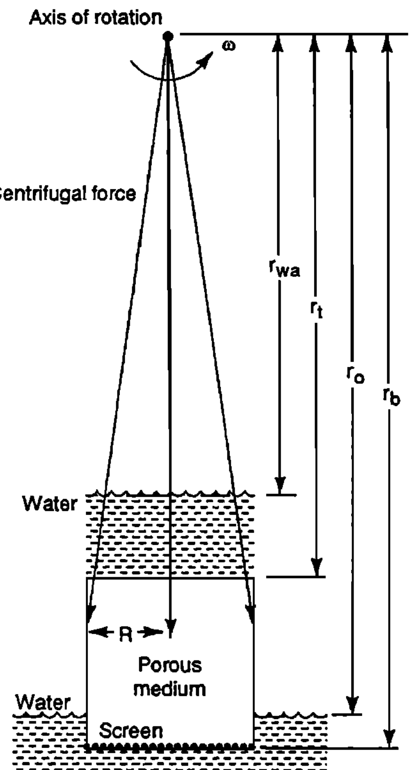

Fig. 1. Cross-sectional diagram of a porous medium in a centrifugal field, with free water establishing hydrostatic pressure boundary conditions. To clarify definitions, the air-water interfaces are shown as flat rather than curved.

$$q = -K_{sat}\left(\frac{dp}{dr} - \rho\omega^2 r\right) \tag{1}$$

where q is the flux density, p is pressure, r is distance from the center of rotation, ρ is the density of water, and ω is angular speed. Using SI units, q would be in meters per second and both dp/dr and $\rho\omega^2 r$ would be in pascals per meter, forcing K to have the unusual units of (m/s)/(Pa/m). For K to be expressed in meters per second (actually (m/s)/(m water/m)), the pressure gradient can be converted to g (taken to have dimensions of force per unit volume and also equivalent to m water/m) with the conversion factor 9.80665 (kPa/m)/g.

Integration of (1) over the whole sample, from r_b to r_t, eliminates the r dependence of p:

$$\int_{p_b}^{p_t} dp = \int_{r_b}^{r_t} (\rho\omega^2 r - q/K_{sat})\, dr. \tag{2}$$

The pressure p_t at r_t results just from the water above the top of the sample. The hydrostatic pressure distribution in a centrifugal field gives p_t as

$$p_t = (\rho\omega^2/2)(r_t^2 - r_{wa}^2). \tag{3a}$$

For the same reasons the pressure at r_b is

$$p_b = (\rho\omega^2/2)(r_b^2 - r_0^2). \tag{3b}$$

Substitution of (3) into (2) and simple analytical integration yields an expression for K:

$$K = \frac{2qL}{\rho\omega^2(r_0^2 - r_{wa}^2)} \tag{4}$$

where L equals $r_b - r_t$, the height of the sample. Sometimes it is also convenient to consider the effective driving force F, the ratio of q to K:

$$F = \frac{\rho\omega^2(r_0^2 - r_{wa}^2)}{2L}. \tag{5}$$

Falling Head

The falling-head centrifuge method (abbreviated FC) is similar to the CC method, but it allows the water level above the soil to fall without replenishment. Measurements of r_{wa} as it increases with time indicate K. Such measurements can be made by weighing the reservoir or by determining the distance $r_{wa} - r_t$ using a depth gauge.

Assuming that water is incompressible and that it does not enter or leave the system, the instantaneous flux density depends on r_{wa} according to geometrical considerations expressed by

$$q = \frac{a}{A}\frac{dr_{wa}}{dt} \tag{6}$$

where A and a are the cross-sectional areas of the sample and falling-head reservoir, respectively. Substitution of (6) into (4) gives a relation between K and r_{wa}:

$$K = \frac{2aL\; dr_{wa}/dt}{A\rho\omega^2(r_0^2 - r_{wa}^2)} \tag{7}$$

Equation (7) applies only on an instantaneous basis, but its time dependence is easy to eliminate by integrating from time t_i to t:

$$\int_{t_i}^{t} dt = \frac{2aL}{A\rho\omega^2 K}\int_{r_i}^{r_{wa}} \frac{dr_{wa}}{r_0^2 - r_{wa}^2} \tag{8}$$

where r_i corresponds to the water height at t_i. After analytical integration and rearrangement, (8) becomes

$$K = \frac{aL}{Ar_0\rho\omega^2(t - t_i)} \log\left[\frac{(r_0 + r_{wa})(r_0 - r_i)}{(r_0 - r_{wa})(r_0 + r_i)}\right]. \tag{9}$$

This formula gives a value of K from measured r_{wa} versus t data and known experimental parameters.

Under some conditions the arrangement diagrammed in Figure 1 may lead to a loss of saturation within some portion of the sample. This effect is possible in a centrifuge where the body force increases with r, even though it would not happen in a uniform gravitational force field. The appendix explains how the possibility of unsaturation can be avoided with attention to the applied boundary conditions.

For a real apparatus the system is not strictly one-dimensional as has been assumed in derivations so far. If the sample is cylindrical, the main non-one-dimensional effect is a curvature of the air-water interface which increases the depth and hence the pressure at the outer edges. Thus the true effective driving force is greater than calculated above, especially when free drainage is permitted at r_b. *Nimmo et al.* [1987] evaluated the results of this effect on unsaturated

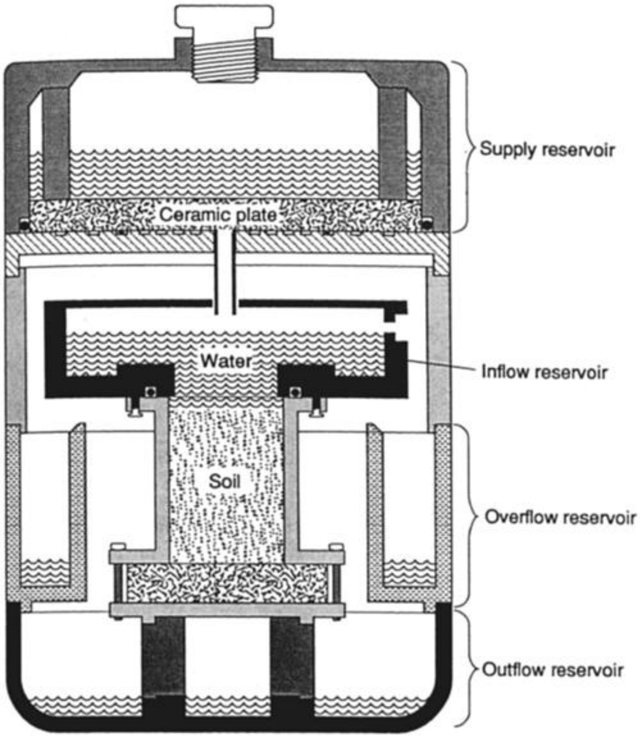

Fig. 2. Cross-sectional view of the apparatus in a centrifuge bucket, configured for constant-head measurements with an "S" retainer. For falling-head measurements the supply reservoir is not used, and all overflow holes of the inflow reservoir are plugged. For falling-head gravity measurements the inflow reservoir was replaced by an adapter plate with tube connector.

flow. The present case is much simpler because there can be no change of moisture content within a saturated sample, so nonuniformity of pressure does not cause nonuniformity of K. Under these conditions, geometrical considerations alone determine the correction factor

$$c = 1 + \frac{R^2}{4(r_b^2 - r_{wa}^2)} \qquad (10)$$

for the $p_b = 0$ case, where R is the radius of the cylindrical sample. Multiplying by c corrects the value of driving force from (5), and dividing by c corrects the value of K_{sat} from (4) or (9). For a typical apparatus, c differs from unity by 1% or less. For the case of $r_0 < r_b$ the curvature of the lower water surface largely compensates for that of the upper, leading to an even smaller correction factor.

TESTING OF METHODS

For testing the new centrifugal methods and comparing them to the ordinary falling-head gravity (FG) method, two different sets of apparatus were used. One set, pictured in Figure 2, was designed specifically for using all three methods on the same sample with a minimum of alterations. This apparatus is designated "S" because it has a small sample size (2.54-cm diameter). The other apparatus was designed for compatibility with equipment used by *Nimmo et al.* [1987] and *Nimmo and Akstin* [1988] for unsaturated SSCM measurements. It is designated "L" because of its relatively large 5.00-cm-diameter samples.

The S apparatus has the soil in a rigid one-piece stainless steel retainer. For adjustment of the water level for the CC method the water reservoir above the soil has overflow ports at different heights. All holes are plugged except the one at the designated height. Water passing through the soil goes through or around the ceramic plate, which serves to hold the soil in place and which has a K_{sat} of about 10^{-7} m/s. The apparatus sits in an outflow-collecting reservoir in the bottom of the centrifuge bucket. For CC use the bucket also contains a supply reservoir and overflow reservoir. The supply reservoir feeds water through the hole in the top of the water reservoir in Figure 2 at a rate somewhat greater than the flow rate through the soil. This imbalance of flow rates keeps the water level at the height of the lowest open overflow port. The overflow reservoir collects the water that emerges from the port. For FC use the supply reservoir is eliminated and all overflow ports are plugged.

The L apparatus also has a one-piece stainless steel soil retainer. The water reservoir attached to the top of the retainer is a 4.5-cm length of acrylic tubing with 5.1-cm inside diameter and 0.6-cm wall. Room temperature vulcanizing rubber (General Electric Company) seals the reservoir to the retainer. (The use of trade names is for identification purposes only and does not imply endorsement by the U.S. Geological Survey (USGS)). This apparatus was used for FC and, with limited success, FG measurements.

The FG tests were conducted, for both S and L apparatus, by replacing the reservoir above the sample with a cap attached to a flexible tube. Water was supplied from a 10-mL buret at a head of about 2 m. These tests followed the method and formulas described by *Klute and Dirksen* [1986]. Corrections were made for water lost by evaporation during the measurements.

The comparison of methods required test media that meet several criteria. (1) The media had to be particulate in order to fill the rigid retainers without significant gaps at the inner wall. (2) They had to have a value of K_{sat} that is measurable by all three methods, say about 10^{-10} m/s. While the CC and FC methods could easily handle much lower values, the FG method would then entail much greater experimental error so that the comparison would be of little value. (3) They could not be excessively compressible, so that the centrifugal force would not significantly affect K_{sat}.

The natural porous media chosen for these tests were fine-textured core samples from great enough depth that the overburden produced a minimally structured medium that is not overly sensitive to additional compressive force. Samples with high clay content were obtained from 5 to 8 m depth at a site on the grounds of the USGS Western Region Headquarters at Menlo Park, California. Another set of suitable core samples, with high silt content, was obtained from 15 to 20 m depth in a region of Athena silt loam on the Palouse Conservation Field Station near Pullman, Washington, operated by the U.S. Department of Agriculture, Agricultural Research Service, in cooperation with Washington State University.

Samples were taken with a ring-lined split-tube sampler. The 6.3-cm-diameter samples were recored into the S and L retainers using a motorized extrusion device. Three of the Menlo Park samples were put into S retainers designated S1, S2, and S3. One additional Menlo Park sample was put into an L retainer, L1, and two Palouse samples were put into retainers L2 and L3. All samples were obtained in an unsaturated state. Some attempts were made to flush with carbon dioxide before saturating, though the effectiveness of this measure in assuring total saturation is dubious because of the small pore sizes and the presence of water in the samples. The samples contained significant amounts of trapped air which probably remained during measurements.

Although the main purpose was to test different methods using natural materials, ceramic materials were also used because they introduce fewer complicating factors. Some of the most important influences are (1) that ceramic is effectively incompressible, (2) that ceramic can be completely saturated without disturbing its structure, and (3) that ceramic is more uniform than soil core samples. To eliminate possible leakage down their walls, the ceramic plates were coated with epoxy. The epoxy (Devcon Corporation, "2-Ton Epoxy") was cast in a Teflon mold, and the excess was then trimmed in a lathe. Two replicate plates, denoted C1 and C2, were made of ceramic with a nominal 5-bar air entry value (Soilmoisture Equipment Corporation) to an effective diameter of 4.85 cm and a thickness of 0.90 cm. For the FC and CC tests the ceramic plates were used in the same apparatus as had been run with the natural media, but the soil core was replaced by an aluminum spacer. Each ceramic plate was placed on a support plate on top of the spacer. The FG test was performed in the same way as for the soil samples.

Measurements for the comparison tests were conducted over a 3-month period during intervals when needed equipment was not in use for other studies. Each sample was initially subjected to the CC or FC method in order that irreversible compaction would occur before, rather than after, the FG tests. For each test the water used was selenate solution (0.01 N $CaSO_4$ and 0.01 N $CaSeO_4$). This solution was designed to minimize changes in K_{sat} by inhibiting microbiological growth and preventing dispersion of clays.

In making the CC tests the apparatus was run for various time intervals in the centrifuge, weighing certain components between each set of runs. The soil and inflow reservoir in Figure 2 were weighed to check the steadiness of the water level above the soil. The change in weight of the outflow reservoir below the soil was taken as an indication of the flux. After a series of runs the volume of water Δw of each run was plotted against the duration Δt of the run, as in Figure 3. A constant K value results in a straight line. In many cases the first one or two runs do not line up well with the others as a result of compaction, air-bubble dissolution, or other effects. These points are discarded, and a linear regression is computed for the others. The line is expected to have a slight positive Δw-intercept as a result of the small amounts of flow that occur just before and just after each centrifuge run. The slope of the line divided by the sample area A is taken as the optimum value of q for use in (4).

Each FC measurement of K also involved a series of runs of various lengths but without replenishment of water after the start. Measurement of r_{wa} after each run gives a set of (t, r_{wa}) pairs, where t is the cumulative time of centrifugation. It is possible to compute K_{sat} using any two data points in the FC formula (9). A better procedure, though, is to use the whole data set, fitting (9) to the data with K_{sat} as a single adjusted parameter. Figure 4 shows two examples of such least squares fits. As in the CC case, the first point or two may not fit with the others, in which case the optimum K_{sat} is taken from a fit to the other points.

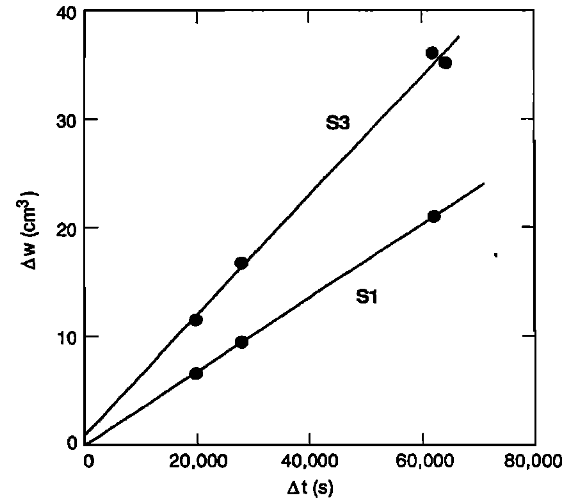

Fig. 3. Data from selected constant-head centrifuge runs. The volume of water Δw passing through the sample during the run is plotted against the time duration Δt of the run.

Results of the K_{sat} measurements are plotted against driving force in Figure 5 for the natural media and in Figure 6 for the ceramics. FG measurements are absent for samples L2 and L3 because their retainers did not allow a leak-free seal to the FG adapter and for C1 because it broke before the measurement sequence was completed. A linear regression, to the CC and FC points only, is shown to indicate trends with force that may exist.

DISCUSSION AND CONCLUSIONS

Analysis of measurement error was based on the combination of estimated random uncertainties and systematic errors from known factors, including uncertainty in apparatus and sample dimensions, the density of water, the centrifuge speed, the measured flow, and the run duration. Estimates of relative error for the natural media were 6.5, 4, and 4% for the CC, FC, and FG methods, respectively. For the CC method the dominant source of uncertainty was imper-

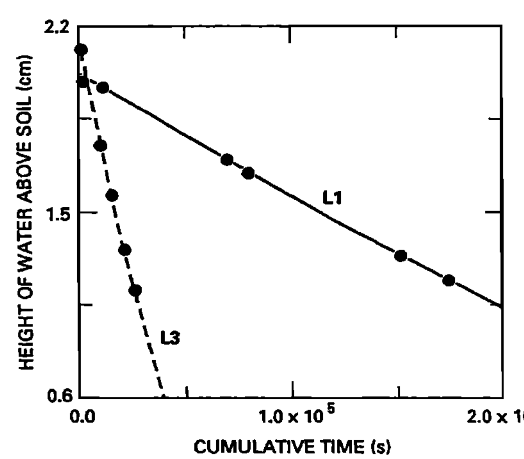

Fig. 4. Data from selected falling-head centrifuge runs. The height of water above the soil ($r_f - r_{wa}$) is plotted against the cumulative elapsed time of two series of runs. The curves are least squares fits of a rearrangement of the FC formula (9) to all points except those at $t = 0$.

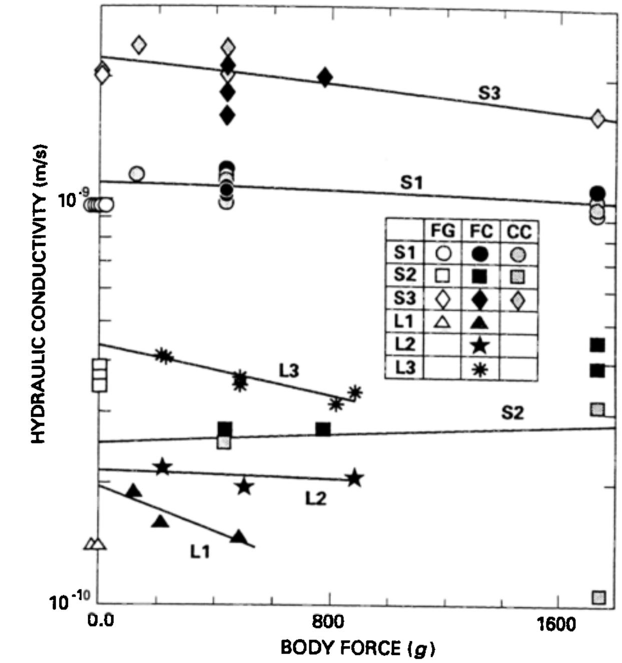

Fig. 5. Comparison of K_{sat} values from three methods of measurement, plotted against the body force to which the midpoint of the sample was subjected during the measurement, for six natural media samples.

fect steadiness of flow rate. For the FC and FG methods no single error source was strongly dominant, but uncertainty in L was the biggest contribution to error. For the ceramics, uncertainty in L was negligible, reducing the respective error estimates to 5.5, 2, and 2%.

The spread in K_{sat} values for each sample is not the result of measurement error alone; the actual saturated conductivity of each sample can change from one measurement to the next. This problem especially affects the CC and FC data with the S apparatus because for these the apparatus is disassembled after each measurement and the sample is exposed, for a few hours or days, to an environment somewhat different from that of the assembled apparatus.

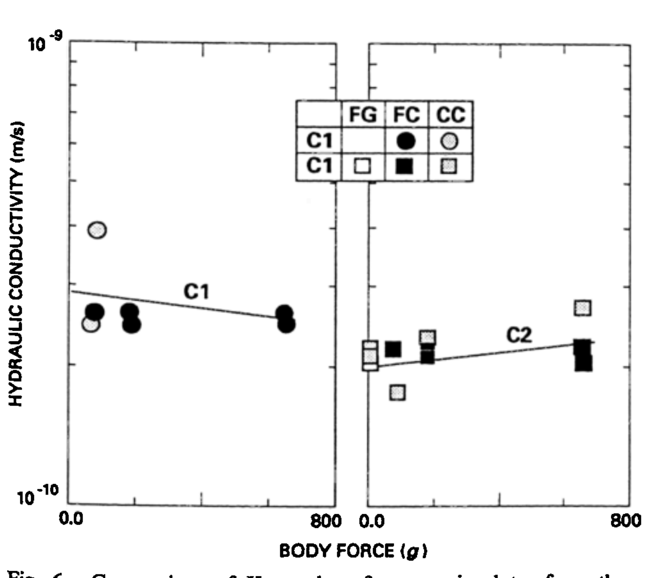

Fig. 6. Comparison of K_{sat} values for ceramic plates from three methods of measurement. Axes have the same scale as Figure 5.

For FG measurements, on the other hand, all equipment remains fully assembled, and one measurement starts right after another ends. Another consideration is that sample characteristics may have changed somewhat over the 3-month duration of testing.

As driving force increases, it might be expected that the measured K_{sat} would decrease as the result of pore reduction from elastic compression. Some evidence for this effect is apparent in Figure 5. For samples with a tighter spread of points at a given force (e.g., L3) the slope of the regression lines is clearly negative. It is not possible here to distinguish between compaction effects and deviations from Darcy's law, but given the small magnitude of the slopes (typically −0.02% for each g increase in force), the results suggest that neither of these effects substantially influences the results. The ceramic plate results in Figure 6 show little variation with body force over the limited range of force covered by these measurements.

In three out of four cases in Figure 5 the FG measurements have lower K_{sat} values than expected from the extrapolation of CC and FC results. For the C2 ceramic case there is no significant difference between the FG and the other methods. This suggests that for the natural media there is a minor systematic influence that causes centrifugally measured K_{sat} values to be slightly greater than expected. Of many possibilities, the most likely are (1) that trapped air pockets might be reduced in size by centrifugal pressures, (2) that elastic compression might reduce L by an unknown amount that might be significant (if Young's modulus is 20 MPa, L would decrease by about 0.03 cm at 400 g), (3) that evaporation from the soil or reservoirs during or between centrifuge runs might be greater than estimated, and (4) that the assumed lower boundary condition may be inexact. More important than slight trends, however, is the fact that within reasonable limits (about 20%) all three methods agree.

Comparing the FC and CC methods, there is no systematic difference apparent in either Figure 5 or 6, but as noted in the discussion of experimental error, the FC method is slightly more accurate. The reason for this is the difficulty of maintaining a constant water level in the CC case. This difficulty was greater in these K_{sat} measurements than it normally is for SSCM measurements of unsaturated K, perhaps because the unsaturated case has a ceramic plate on the constant-head reservoir which may exert a stabilizing influence. In the technically simpler case of ceramic plates (Figure 6) there is good agreement among all FC measurements. Besides accuracy, the FC method has the advantages of requiring no regulating mechanism and being simpler to operate. A possible disadvantage for some media is that the FC method, like all falling-head methods, requires the assumption that transient effects are negligible. That is, that K_{sat} and water storage in the sample do not change significantly over the range of heads applied. Overall, in most applications the FC method is probably better than the CC method.

Comparing the gravity and centrifuge methods, superficially the FG method might seem better for accuracy, but considering the additional reasons noted above for greater spread in FC and CC measurements, there is little reason for preference on this basis. It should also be noted that the apparatus used in these tests, especially the S version, was far from optimum for FC measurements. A smaller-diameter reservoir above the soil might improve the accuracy and the speed of operation of the FC method, in particular when K_{sat} is very small. Even without modifications, the FC method was easier and faster than the FG method. The use of low centrifuge speeds would easily extend the range of the FC method to higher K_{sat} values, well into the range where FG might normally be the method of choice. The FC method could also be used with extremely simple apparatus, for example, a single partly filled cylinder could serve as a combined soil retainer and reservoir. The FC method has another distinct operational advantage in that the apparatus has few places that are vulnerable to leaks.

In most cases the biggest consideration concerning the use of centrifugal force is that compaction can alter the structure of the material so as to influence K_{sat}. This effect is primarily important for unconsolidated materials, and for the case of highly structured soils it would make centrifugal methods unsuitable. For others, though, such as consolidated materials and densely packed sands, the effect may be negligible. For some of the cases where the effect of compaction is significant, it may be possible to correct for it. Another possibility, however, is to use this effect to advantage in permitting K_{sat} to be measured under conditions of a simulated overburden pressure approximately equal to that of the original location of the sample. At accelerations of a few thousand g, a small weight on top of the sample can exert a mechanical pressure equivalent to that of hundreds of meters of overlying material. To simulate a given overburden, it is better to use a large weight at low speed than a small weight at high speed, for greater uniformity of stress from the top to the bottom of the sample.

In situations where compaction is not a serious problem, such as for samples from considerable depths or consolidated or unstructured materials, centrifugal methods have many advantages: simplicity, ease of operation, reasonable accuracy, minimal time requirements for a large number of measurements, and possibly a very wide range of measurable K_{sat}.

APPENDIX: AVOIDANCE OF UNSATURATION

Because the centrifugal force is greater near the bottom than near the top of the sample, it is possible for lower portions of the sample to desaturate even though water is ponded above the top surface. This problem would most commonly occur in the easily instrumented case of a zero-pressure boundary at the bottom of the sample, when the depth of water above the sample falls below a critical threshold. This appendix presents information for determining this threshold in advance.

The total potential used in Darcy's law (1) may be written as

$$\Phi = p + \Phi_c \tag{A1}$$

where the centrifugal potential is

$$\Phi_c = -\tfrac{1}{2}\rho\omega^2 r^2. \tag{A2}$$

Darcy's law in the form

$$q/K = -d\Phi/dr \tag{A3}$$

makes it clear that $\Phi(r)$ will be linear as long the flow is steady and K is uniform throughout the sample. Figure A1

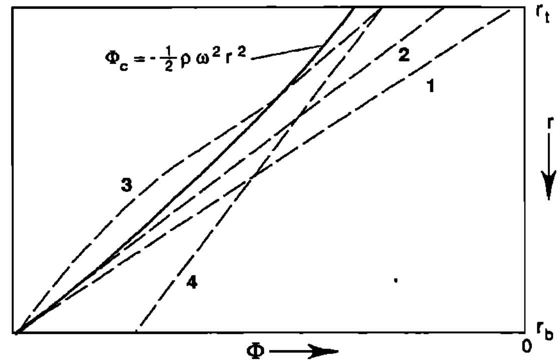

Fig. A1. A typical centrifugal potential Φ_c profile with four possible total potential Φ profiles.

shows a typical example of the parabolic form of $\Phi_c(r)$ with four hypothetical examples of $\Phi(r)$ profiles that are compatible with it.

If the sample starts totally saturated, it is safe to assume that it will remain so if p does not become negative. Equivalently,

$$\Phi \geq \Phi_c. \tag{A4}$$

On a graph such as Figure A1 this means that $\Phi(r)$ does not cross over to the left of $\Phi_c(r)$. The profiles numbered 1–3 illustrate this for the case of $p_b = 0$ and three different p_t values. Profile 1 clearly satisfies the criterion (A4); profile 2, being tangent to $\Phi_c(r)$ at r_b, barely satisfies it; and profile 3 does not satisfy it (and if desaturation occurs, it would in actuality become nonlinear to the left of $\Phi_c(r)$). Profile 4 has the same p_t as profile 3 but remains to the right of $\Phi_c(r)$ because of its positive p_b boundary condition.

Since profile 2 has the minimum p_t to satisfy (A4) with $p_b = 0$, its tangent condition leads to a formula for the minimum p_t. The slope of $\Phi(r)$ at r_b must be less than or equal to that of $\Phi_c(r)$ (noting that both slopes are negative), so that the criterion may be written

$$\left.\frac{d\Phi}{dr}\right|_{r_b} \leq \left.\frac{d\Phi_c}{dr}\right|_{r_b}. \tag{A5}$$

Using the hydrostatic boundary condition of (3a) to obtain $d\Phi/dr$ and differentiating (A2) to obtain $d\Phi_c/dr$, the criterion (A5) is equivalent to

$$\frac{-\frac{1}{2}\rho\omega^2(r_b^2 - r_{wa}^2)}{r_b - r_t} \leq -\rho\omega^2 r_b, \tag{A6}$$

which can be algebraically reduced to

$$r_{wa} \leq (2r_b r_t - r_b^2)^{1/2}. \tag{A7}$$

The maximum r_{wa} to assure that nonnegative p is independent of speed and medium. Typical values of 20 cm for r_b and 16 cm for r_t produce a maximum r_{wa} of 15.49 cm, indicating that the water surface must be kept more than 0.51 cm above the top of the sample.

Acknowledgments. We are grateful to David Stonestrom, who conducted the drilling and sampling operations in which the core samples were obtained, and to the USGS-WRD Pacific Northwest

District, whose assistance and equipment were indispensible in obtaining the samples from the Washington site. We are also indebted to Dale Hammermeister, who developed the first version of the constant-head centrifuge method.

REFERENCES

Alemi, M. H., D. R. Nielson, and J. W. Biggar, Determining the hydraulic conductivity of soil cores by centrifugation, *Soil. Sci. Soc. Am. J.*, *40*, 212–218, 1976.

Bianchi, W. C., and E. E. Haskell, Jr., A strain gage pressure cell for rapid determination of hydraulic conductivity of soil cores, *Proc. Am. Soc. Test. Mater.*, *63*, 1227–1234, 1963.

Brace, W. F., J. B. Walsh, and W. T. Frangos, Permeability of granite under high pressure, *J. Geophys. Res.*, *73*(6), 2225–2236, 1968.

Klute, A., and C. Dirksen, Hydraulic conductivity and diffusivity: Laboratory methods, in Methods of Soil Analysis, vol. 1, edited by A. Klute, *Agronomy*, *9*, 687–732, 1986.

Mundell, J. D., and B. Bailey, The design and testing of a compacted clay barrier layer to limit percolation through landfill covers, in Hydraulic Barriers in Soil and Rock, edited by A. J. Johnson et al., *ASTM Spec. Tech. Publ.*, *STP 874*, 246–262, 1984.

Neuzil, C. E., Groundwater flow in low-permeability environments, *Water Resour. Res.*, *22*, 1163–1195, 1986.

Nimmo, J. R., and K. C. Akstin, Hydraulic conductivity of a sandy soil at low water content after compaction by various methods, *Soil Sci. Soc. Am. J.*, *52*, 303–310, 1988.

Nimmo, J. R., J. Rubin, and D. P. Hammermeister, Unsaturated flow in a centrifugal field: Measurement of hydraulic conductivity and testing of Darcy's law, *Water Resour. Res.*, *23*, 124–134, 1987.

Ohle, E. L., The influence of permeability on ore distribution in limestone and dolomite, I, *Econ. Geol.*, *46*(7), 667–706, 1951.

Rubin, J., D. P. Hammermeister, and J. R. Nimmo, Method and apparatus for steady-state measurement of liquid conductivity in porous media, *U.S. Patent 4,679,422*, U.S. Patent and Trademark Off., Arlington, Va., 1987.

Terzaghi, C., Principles of soil mechanics, III, Determination of permeability of clay, *Eng. News Rec.*, *95*(21), 832–836, 1925.

K. A. Mello and J. R. Nimmo, Water Resources Division, U.S. Geological Survey, 345 Middlefield Road, MS 421, Menlo Park, CA 94025.

(Received March 20, 1990;
revised January 31, 1991;
accepted February 1, 1991.)