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Soil water retention and maximum capillary drive from saturation to oven dryness

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Abstract. This paper provides an alternative method to describe the water retention curve over a range of water contents from saturation to oven dryness. It makes two modifications to the standard *Brooks and Corey* [1964] (B-C) description, one at each end of the suction range. One expression proposed by *Rossi and Nimmo* [1994] is used in the high-suction range to a zero residual water content. (This Rossi-Nimmo modification to the Brooks-Corey model provides a more realistic description of the retention curve at low water contents.) Near zero suction the second modification eliminates the region where there is a change in suction with no change in water content. Tests on seven soil data sets, using three distinct analytical expressions for the high-, medium-, and low-suction ranges, show that the experimental water retention curves are well fitted by this composite procedure. The high-suction range of saturation contributes little to the maximum capillary drive, defined with a good approximation for a soil water and air system as $H_{cM} = \int_0^\infty k_{rw}\, dh_c$, where k_{rw} is relative permeability (or conductivity) to water and h_c is capillary suction, a positive quantity in unsaturated soils. As a result, the modification suggested to describe the high-suction range does not significantly affect the equivalence between Brooks-Corey (B-C) and *van Genuchten* [1980] parameters presented earlier. However, the shape of the retention curve near "natural saturation" has a significant impact on the value of the capillary drive. The estimate using the Brooks-Corey power law, extended to zero suction, will exceed that obtained with the new procedure by 25 to 30%. It is not possible to tell which procedure is appropriate. Tests on another data set, for which relative conductivity data are available, support the view of the authors that measurements of a retention curve coupled with a speculative curve of relative permeability as from a capillary model are not sufficient to accurately determine the (maximum) capillary drive. The capillary drive is a dynamic scalar, whereas the retention curve is of a static character. Only measurements of infiltration rates with time can determine the capillary drive with precision for a given soil.

1. Introduction

In this paper we are trying to address several issues related to (1) the manner in which soil water retention data are fitted to a priori analytical expressions and (2) the inferences that may be drawn from these curves in conjunction with theoretical relative permeabilities, for example, in their use for prediction of infiltration rates. We divide values of suction into three ranges: the high-suction range (from oven dryness to approximate field capacity), the middle range (field capacity to roughly air entry pressure), and the low range (entry pressure to zero suction). Different expressions are needed for each range.

We recommend changes to overcome some difficulties associated with the *Brooks-Corey* [1964] expressions. For the high-suction range a slightly modified version of an exponential expression previously used by *Rossi and Nimmo* [1994] (R-N) is selected. For the low-suction range a simple, new algebraic relation is adopted, requiring that it meet certain conditions of continuity and smoothness (i.e., continuity of slope) where it reattaches with the traditional *Brooks and Corey* [1964] (B-C) expressions for the middle range. At low suction the B-C expressions are sometimes inconvenient, lacking a smooth, gradual transition to zero suction, which may not be realistic physically for soils that do not display a clear entry pressure. It also creates numerical difficulties with computational algorithms. Thus some investigators [e.g., *Morel-Seytoux and Billica*, 1985; *Nofziger et al.*, 1989; *Touma*, 1984; *White et al.*, 1992] prefer to use the *van Genuchten* [1980] (vG) or other smoother descriptions.

Because many soils in the past have been characterized strictly with the B-C approach [e.g., *Rawls and Brakensiek*, 1989], using this new procedure may create practical problems. However, if the original data are still available, one can fit them to the new expressions. Where the original data are not available, one can attempt to develop a correspondence between the "old" B-C parameters and the new parameters introduced here. Through this equivalence the information about these soils is retained and can possibly be enhanced.

There is no guarantee that in combination with theoretical relative permeabilities the various curve-fitting approaches (B-C, vG, R-N, or the proposed approach), even when based

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on the original data, will automatically provide the same value for the "capillary drive" [*Morel-Seytoux et al.*, 1996]. For this reason a fundamental constraint for the development of an equivalence is that the value of the capillary drive be preserved in the process. With the proposed approach, equivalences can be derived that will preserve that quantity. Although the capillary drive may be preserved in the equivalences, there is no guarantee that its determined value, based on one approach or another, will be correct.

To elaborate, one of the difficulties with the Brooks-Corey expression for the water retention curve is that it is only applicable for relatively low to moderate suctions. *Brooks and Corey* [1964, p. 17] stated explicitly that "A model designed in accordance with the theory presented here must be restricted in its applications to systems that do not reach saturations less than S_r (residual saturation), because the theory is applicable only to saturations greater than S_r." To determine the best parameters to fit the data, the B-C method considers the so-called "residual water content" as a parameter whose calibrated value is rarely zero. Yet the "true" residual water content at oven dryness is actually zero.

Rossi and Nimmo [1994] showed that at very low water contents, experimental suction data could be well fitted by an exponential law. They proposed to fit the entire range of water contents from saturation to oven dryness by using three expressions: (1) an exponential form to fit the data at very high suctions; (2) the B-C power law in the middle range (assuming systematically, a zero residual water content); and (3) the *Hutson and Cass* [1987] parabolic curve close to saturation. They required that the expressions match smoothly at junction points. If actual data are available, one can then follow the procedures of their research to calibrate their model on the data. In many cases the original data are not available, but B-C parameters are. Thus, how does one use B-C parameters and extrapolate beyond the calibrated residual water content to oven dryness? In the original R-N procedure the residual water content is assumed to be zero. Consequently, the exponents in the power law used for the middle range in the B-C and R-N fits, in general, will usually not have the same value. To retain the information on soils contained in the B-C parameters fitted to them, it is more efficient to modify the R-N procedure by using the concept of an "artificial" residual saturation in the middle range, though it has no significance outside that range.

If the Brooks-Corey parameters are known, it is easy to determine the value of the maximum capillary drive [*Morel-Seytoux et al.*, 1996]. Thus, would the exponential extension in the high-suction range have an impact on the estimation of this capillary drive? If the impact is insignificant, one can define the capillary drive in terms of a set of "old" B-C parameters compatible with the exponential extrapolation to oven dryness, regardless of the actual value of the residual water content. We show that this is the case, and thus the "old" B-C parameters can be used in conjunction with the exponential extrapolation to oven dryness.

Finally, the main consideration for not using the *Hutson and Cass* [1987] low-suction expression was because it requires an integration that cannot be obtained in explicit analytical form. In addition, the new expression is more flexible, being cubic rather than parabolic.

A new procedure was introduced to fit the entire range of suctions which uses the exponential form for very high suctions and a B-C power law in the middle range. However, it does not assume systematically a zero residual water content, and at low suctions a new expression is introduced. With that expression the capillary drive is obtained in closed explicit form.

2. Objectives

The primary objective of this research is to provide a simple way to convert Brooks-Corey (B-C) (and others, such as van Genuchten) parameters to another set of parameters associated with a modified version of earlier research [*Rossi and Nimmo*, 1994] and vice versa for use in situations where saturated and very dry conditions are likely to be encountered. For example, the proper description of the soil properties at high and low water content is crucial, such as in continuous watershed modeling and capillary barrier design. Important in this conversion is preservation of the maximum value of a well-defined physical characteristic, the "(effective) capillary drive" [*Morel-Seytoux and Khanji*, 1974, 1975]. A good approximation for a soil water and air system is

$$H_{cM} = \int_0^\infty k_{rw}\, dh_c \qquad (1)$$

where k_{rw} is relative permeability (or conductivity) to water and h_c is capillary suction, a positive quantity, the negative of the matric head. The physical significance of H_{cM} has been discussed previously [*Morel-Seytoux et al.*, 1996].

Another objective of this research is to clarify the accuracy of estimation of the capillary drive regarding the chosen expressions to fit the retention curve and derived expressions for relative permeability. The same set of retention data fitted to different expressions, be they B-C or vG or others, will not yield generally the same value for the capillary drive. However, if the equivalence between parameters for the B-C and vG or other approaches is defined in a way that preserves the value of the capillary drive [*Morel-Seytoux et al.*, 1996] naturally, by definition all expressions will yield the same value although there is no guarantee that this value is correct.

3. Brooks-Corey (B-C) and Rossi-Nimmo (R-N) Expressions

3.1. Soil Hydrologic Characteristics: Brooks-Corey Expressions

Normalized water content is defined as

$$\theta^* = (\theta - \theta_r)/(\tilde{\theta} - \theta_r) \qquad (2)$$

where θ is (volumetric) water content, $\tilde{\theta}$ is water content at natural saturation (i.e., porosity minus trapped air content), and θ_r is "residual water" content. No physical significance is attached to this residual water content. It is purely a parameter chosen to improve the fit of the data to the equation. The B-C expressions are

$$h_c = h_{ce}\theta^{*-M} \qquad h_c \geq h_{ce} \qquad (3)$$

$$k_{rw} = \theta^{*p} \qquad h_c \geq h_{ce} \qquad (4a)$$

$$k_{rw} = 1 \qquad h_c \leq h_{ce} \qquad (4b)$$

No physical significance is attached to the parameter h_{ce} even though it is herein called entry pressure. For infiltration problems the retention curve must be the wetting one. To reduce the number of parameters to two, a relation developed by *Corey* [1977] is used

$$p = 3 + 2M \quad (5a)$$

or

$$M = (p - 3)/2 \quad (5b)$$

In Corey's notation a primary parameter λ (called "pore size index") is used, that is, the inverse of M; here as in the earlier paper [*Morel-Seytoux et al.*, 1996], we prefer to use p (the exponent in the power law of normalized water content for relative permeability) as the primary parameter. Performing the integration in (1), the effective capillary drive is

$$H_c = \frac{h_{ce}}{\alpha - 1}\left[\alpha - \left(\frac{h_{ci}}{h_{ce}}\right)^{-\alpha+1}\right] \quad (6)$$

where $\alpha = p/M$ and h_{ci} is the initial value of capillary pressure in the soil before it is wet to saturation with zero suction at the soil surface. The maximum value of the capillary drive is

$$H_{cM} = \alpha h_{ce}/(\alpha - 1) \quad (7)$$

Except for notation this is the expression presented by *Brakensiek* [1977].

3.2. Soil Hydrologic Characteristics: Rossi-Nimmo Expressions

Having defined (effective) saturation as

$$S_e = \theta/\tilde{\theta} \quad (8)$$

the Rossi-Nimmo junction model expressions are (except for some change in notations)

$$h_c = h_{cD}e^{-a_{RN}S_e} \qquad h_c \geq h_{cmE} \quad (9)$$

$$h_c = h_{ce}S_e^{-M} \qquad h_{ce} \leq h_c \leq h_{cmE} \quad (10)$$

where a_{RN} is a parameter to be determined by fitting to observed data, the subscripts $_{mE}$ refer to a matching point at which the two expressions must be continuous in value and slope, and h_{cD} is oven dry suction. For relative permeability, again using (4), the expressions are much the same as in the B-C case except that the argument in the power law is the saturation as defined by (8) thus

$$k_{rw} = S_e^p \qquad h_c \geq h_{ce} \quad (11a)$$

$$k_{rw} = 1 \qquad h_c \leq h_{ce} \quad (11b)$$

The requirements that the two expressions given by (9) and (10) must be continuous and smooth at the matching point lead to the relations between the parameters:

$$h_{cmE} = h_{cD}e^{-M} \quad (12)$$

$$S_{em} = \left(\frac{h_{cmE}}{h_{ce}}\right)^{-1/M} \quad (13)$$

$$a_{RN}S_{em} = M \quad (14a)$$

or

$$a_{RN} = M/S_{em} \quad (14b)$$

For these relations to hold, the residual water content must be assumed to be zero in the B-C expressions. In the Rossi-Nimmo (R-N) case, since there are two expressions for capillary pressure, the integration for the capillary drive is carried out by parts for the two separate zones, and the result for its maximum value is

Table 1. Summary of Soil Parameters Determined by Fitting

Soil Name	Soil Number	$\tilde{\theta}$	B-C λ	h_{ce}, cm
Palouse	1	0.44	0.25	43.4
Palouse B	2	0.55	0.16	16.7
Walla Walla	3	0.39	0.28	44.6
Salkum	4	0.48	0.29	131.2
Royal	5	0.35	0.41	53.8
L-soil	6	0.18	0.33	13.3
Rothamsted	7	0.51	0.34	176.3

Data are after *Rossi and Nimmo* [1994].

$$H_{cM} = \frac{h_{ce}}{\alpha - 1}\left[\alpha - \left(\frac{h_{cmE}}{h_{ce}}\right)^{-\alpha+1}\right] + h_{cD}\frac{p!}{a_{RN}^p}\left[(1 - e^{-M}) - e^{-M}\sum_{k=0}^{p-1}\frac{M^{p-k}}{(p-k)!}\right] \quad (15)$$

an expression only valid for p integer. Numerically, it is easier to calculate H_{cM} through an iterative procedure, namely,

$$H_c(k) = \frac{k}{a_{RN}}H_c(k - 1) - h_{cmE}(S_{em})^k \qquad k = 1, \cdots, p \quad (16)$$

with

$$H_c(0) = h_{cD}(1 - e^{-M}) \quad (17a)$$

$$H_{cM} = H_c(p) \quad (17b)$$

The parameters in the proposed expressions are a_{RN}, h_{cD}, h_{ce}, p (or M or α), and h_{cmE}. Since there are two equations relating them and if, in addition, one selects for h_{cD} a standard value of 10^7 cm, the number of independent parameters is only two, as in the B-C case.

4. Equivalence Criterion and Impact of Exponential Extension on the Capillary Drive

4.1. Conversion From R-N to B-C Parameters

For simplicity it is assumed the parameters α are the same. Thus, given the value of H_{cM}, as calculated from the (16) and (17), the value of α carries over, and the only parameter to estimate is the entry pressure $(h_{ce})_{\text{B-C}}$, which is calculated as

$$(h_{ce})_{\text{B-C}} = \frac{\alpha - 1}{\alpha}H_{cM} \quad (18)$$

This procedure was tested on seven soils [*Schofield*, 1935; *Campbell and Shiozawa*, 1992; *Rossi and Nimmo*, 1994]. Using the B-C procedure, the calculated value of p for each soil was rounded to the nearest integer; the value of M was recalculated using the relation (5b). Table 1 shows the characteristics of the seven soils with parameters calibrated on the data for the model junction [*Rossi and Nimmo*, 1994, p. 2, Table 3].

Table 2 provides the values of the equivalent $(h_{ce})_{\text{B-C}}$ for the seven soils. All lengths are in centimeters. The values of the computed entry pressures in the two models are essentially the same because the contribution to the total capillary drive from the exponential portion of the retention curve is negligible for

Table 2. Values of the Entry Pressures and Maximum Capillary Drive for the Proposed Model and the B-C Model

M	h_{ce}	$(h_{ce})_{\text{B-C}}$	H_{cM}	$HC_{\exp}$	$(H_{cM})_{\text{MS}}$
4.0	43.40	43.40	68.2	0.0000	68.2
6.2	16.70	16.70	27.8	0.0002	27.8
3.6	44.60	44.60	68.6	0.0000	68.6
3.4	131.20	131.20	201.8	0.0003	201.8
2.4	53.80	53.80	78.3	0.0000	78.3
3.0	13.30	13.30	20.0	0.0000	20.0
2.9	176.30	176.30	264.5	0.0001	264.5

$HC_{\exp}$ is the contribution of the "exponential" part of the expression (low water contents) to the total capillary drive and $(H_{cM})_{\text{MS}}$ is the contribution of the B-C range (medium water contents to natural saturation) to the total capillary drive.

Lengths are given in centimeters.

all soils. Thus, in practice, if one calibrated the data with the R-N model, the parameters M and h_{ce} could be used for the straight Brooks-Corey model (and vice versa), and in the conversion the capillary drive would be preserved.

In the process of conversion the value of a residual water content is not obtained, and it does not affect the value of the capillary drive as the following argument shows. Let the relative permeability and suction be expressed as

$$k_{rw} = X^{p} \tag{19}$$

$$h_c = h_{ce} X^{-M} \tag{20}$$

where X could be normalized water content or effective saturation or another function of water content. Elimination of X between (19) and (20) leads to

$$k_{rw} = (h_c/h_{ce})^{-\alpha} \tag{21}$$

and from the definition of capillary drive

$$H_{cM} = \int_0^{h_{cD}} k_{rw}\, dh_c = h_{ce} + \int_{h_{ce}}^{h_{cD}} \left(\frac{h_c}{h_{ce}}\right)^{-\alpha} dh_c$$

$$= h_{ce}\left\{1 + \frac{1}{\alpha - 1}\left[1 - \left(\frac{h_{cD}}{h_{ce}}\right)^{-\alpha+1}\right]\right\} \tag{22}$$

which shows that (since the result is independent of the choice of X) for a given value of α the value of the capillary drive is unaffected by values of θ_r. The individual curves of relative permeability and suction versus water content will be different, but the curve k_{rw} versus h_c will be the same, which is the curve that matters for the capillary drive and problems of infiltration [*Morel-Seytoux et al.*, 1996]. If necessary, one could use an estimate of residual water content from correlation studies established for many soils [*Rawls and Brakensiek*, 1989].

4.2. Conversion From B-C Parameters to Those Needed in Modified R-N Expressions

Conversion of these parameters is of primary interest, and since the Brooks-Corey procedure has been used for years, many soils have been characterized by the B-C parameters, and many correlations were developed between readily available soil properties such as percentage sand and clay and the Brooks-Corey parameters [*Rawls and Brakensiek*, 1989]. The problem with the B-C procedure as demonstrated by *Rossi and Nimmo* [1994] is that at high suction it does not represent the data well. However, the R-N procedure assumes that residual water content in the B-C expression is zero, but for many soils for which B-C parameters are available, that calibrated value is not zero. Thus it became necessary to modify the R-N procedure to account for this.

5. New Expressions

The soil hydrologic characteristics between high-suction range and oven dryness are the same as in the R-N procedure, namely, (8), (9), and (10), except that the effective saturation is defined in the sense of Corey, that is, including the nonzero S_r. In the middle range the equations are standard B-Cs (equations (2) and (3)), including the definition of effective saturation or normalized water content.

For the soil matching conditions at the junction of the two ranges, let us designate the matching point between the high- and middle-suction ranges as θ_{mE} and its normalized value as θ^*_{mE}. Then the condition of continuous and smooth reattachment requires that the following two equations be satisfied by the two unknowns θ_{mE} and a_{RN} (the parameter in (9), the exponential expression for the suction), namely,

$$a_{RN}(\theta_{mE} - \theta_r) - M\tilde{\theta} = 0 \tag{23}$$

$$a_{RN}\frac{\theta_{mE}}{\tilde{\theta}} - M \ln\left[\frac{(\theta_{mE} - \theta_r)}{(\tilde{\theta} - \theta_r)}\right] - \ln\left(\frac{h_{cD}}{h_{ce}}\right) = 0 \tag{24}$$

These two equations are solved iteratively by linearization using Taylor series expansions and checking the magnitude of the residues in original equations. As a starting point, one uses values that assume residual water content is zero, namely,

$$(\theta_{mE})_I = \tilde{\theta}\left(\frac{h_{cD}e^{-M}}{h_{ce}}\right)^{-1/M} \tag{25a}$$

$$(a_{RN})_I = \frac{M\tilde{\theta}}{(\theta_{mE})_I} \tag{25b}$$

In the low-suction range, let θ_m designate the water content and its normalized value $\theta^*_m = x_m$, where the B-C expression must match the low-suction range expression for suction and let h_{cm} designate the corresponding suction. Then the expression for the low suction range with $\theta^* = x$ is chosen to be of the following form:

$$h_c = h_{cm}\left\{1 - \frac{M(x - x_m)}{x_m} - \frac{[(M+1)x_m - M](x - x_m)^2}{x_m(1 - x_m)^2} + a_{MS}(x - x_m)^2(1 - x)\right\} \tag{26}$$

One can verify at the matching point ($x = x_m$) that the suction is continuous and smooth. Also, we desire the slope of the suction curve at saturation to be more negative than at the matching point. This will be satisfied if $x_m > M/(M + 1)$ and $a_{MS} > 0$. We shall constrain these parameters to satisfy these inequalities. It is also desirable for the two expressions for middle- and low-suction ranges to have the same curvature at the matching point. That requirement will be satisfied if a_{MS} takes the value a^C_{MS}, given by the following expression:

$$a^C_{MS} = \frac{1}{x_m(1 - x_m)}\left\{\frac{M(M+1)}{2x_m} + \frac{[(M+1)x_m - M]}{(1 - x_m)^2}\right\} \tag{27}$$

If we require that the curvature on the low-suction range, as given by the low-suction range expression, be positive, it is necessary that

$$a_{MS} > a_{MS}^{\min} = \frac{[(M+1)x_m - M]}{x_m(1-x_m)^3} \quad (28)$$

Additionally, one does not wish the slope of the suction curve to turn positive. This implies that the discriminant Δ of a second-degree equation never be positive; the resulting inequality to be satisfied is

$$\Delta = [(a_{MS} - a_{MS}^{\min})(1-x_m)]^2 - 3a_{MS}(M/x_m) \le 0 \quad (29)$$

The real root of the equation $\Delta = 0$ for the variable a defines the maximum acceptable value for a such that

$$a_{MS}^{\max} = a_{MS}^{\min} + \frac{3M}{2x_m(1-x_m)}\left\{1 + \sqrt{1 + \frac{4[(M+1)x_m - M]}{3M(1-x_m)}}\right\} \quad (30)$$

We define a_{MS} as

$$a_{MS} = a_{MS}^{\min} + f_{MS}(a_{MS}^{\max} - a_{MS}^{\min}) \quad (31)$$

where f_{MS} varies in the range 0 to 1. The optimal values for f_{MS} and x_m are the ones that minimize the sum of squares of deviations between the measured values and the fitted ones. The algorithm was programmed in a FORTRAN code that is readily available.

6. Results on Seven Data Sets

The new procedure to fit observations was tested on the same data sets used by *Rossi and Nimmo* [1994]. Figures 1 through 7 show the excellent agreement obtained by the procedure. In all figures the fitted curve was only evaluated at water contents for which there were observations. These points are joined by straight lines. The fitted curve at the low-suction range is a third-degree polynomial and thus is much smoother than these lines. Figure 8 displays the fitted curve to the data for soil 6, the L soil. Figure 8 shows the "matching" points that indicate the passage from one expression to the next. Table 3 provides measures of goodness of fit for the three ranges of suction for the seven soils. For the high- and low-suction range

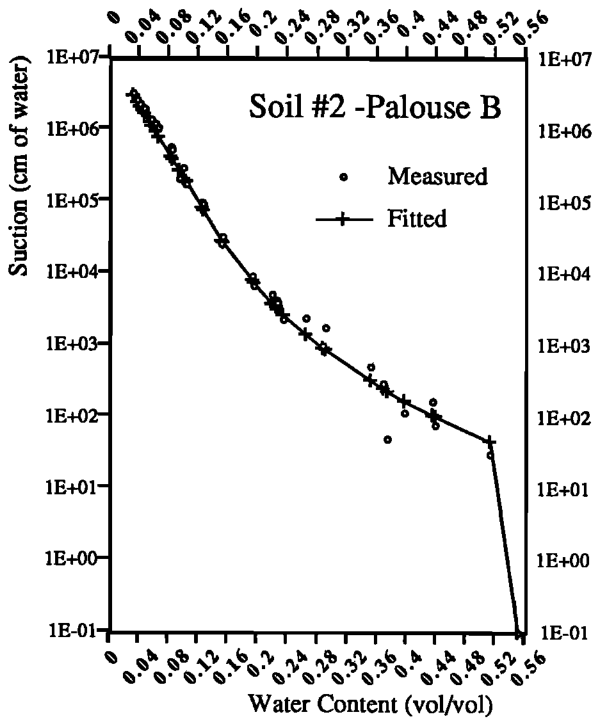

Figure 2. Comparison of measured versus fitted values of retention curve for soil 2.

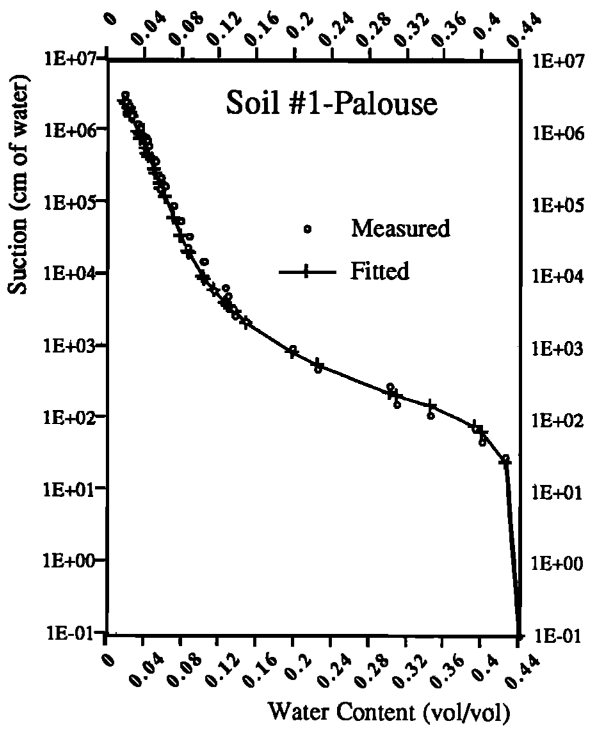

Figure 1. Comparison of measured versus fitted values of retention curve for soil 1.

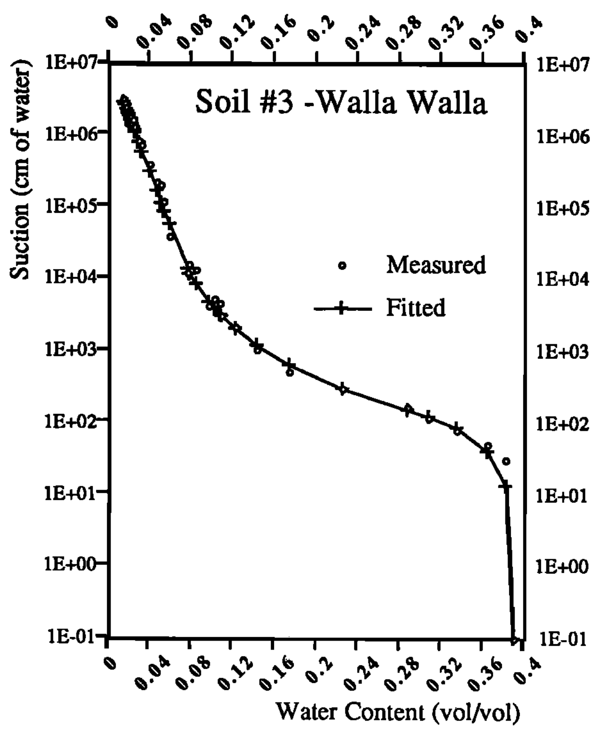

Figure 3. Comparison of measured versus fitted values of retention curve for soil 3.

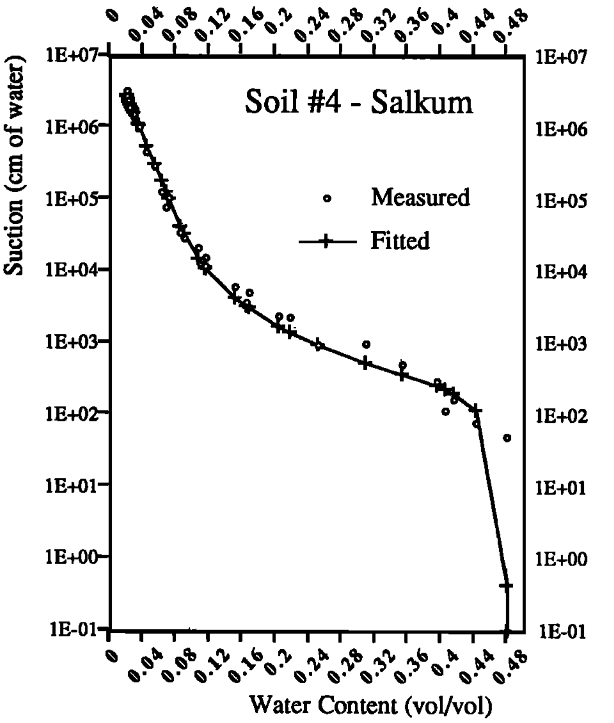

Figure 4. Comparison of measured versus fitted values of retention curve for soil 4.

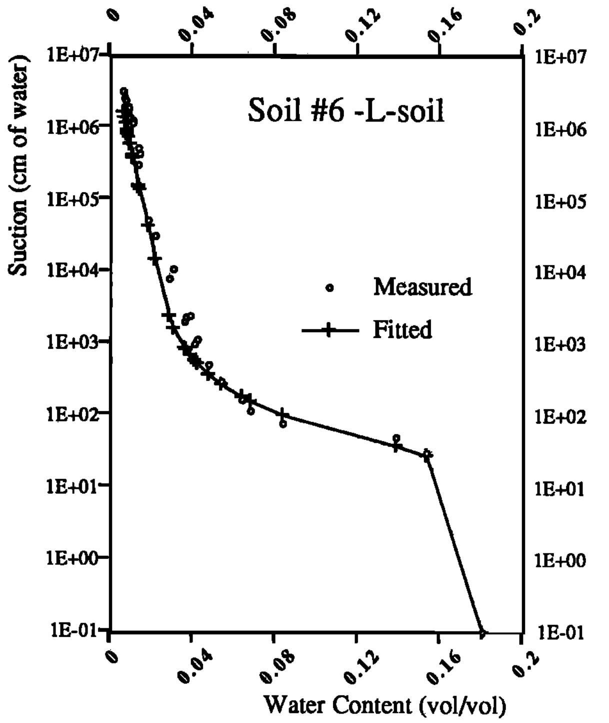

Figure 6. Comparison of measured versus fitted values of retention curve for soil 6.

the measure was defined as the square root of one minus a ratio. The numerator of the ratio is the sum of squares of the differences between observed and fitted values, and the denominator is the variance of the observations. For the middle range (B-C expression) it is given by the absolute value of the coefficient of correlation of the regression of the logarithms of suction versus the logarithms of normalized water content. The statistical measures confirm the good fit as visually demonstrated in all figures. Table 3 also provides values of the parameters for different parts of the retention curve.

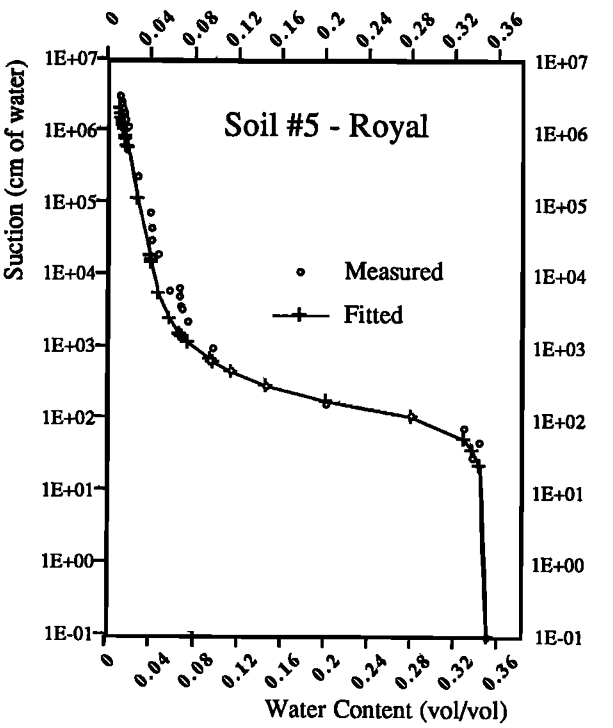

Figure 5. Comparison of measured versus fitted values of retention curve for soil 5.

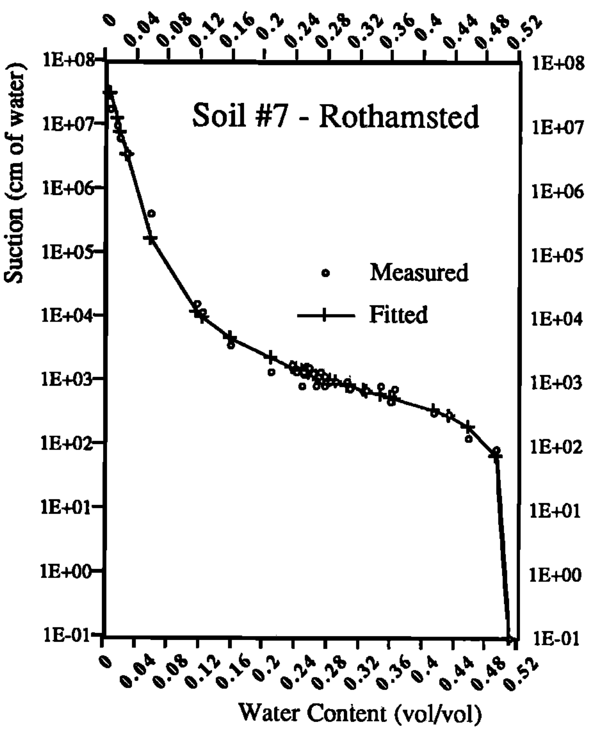

Figure 7. Comparison of measured versus fitted values of retention curve for soil 7.

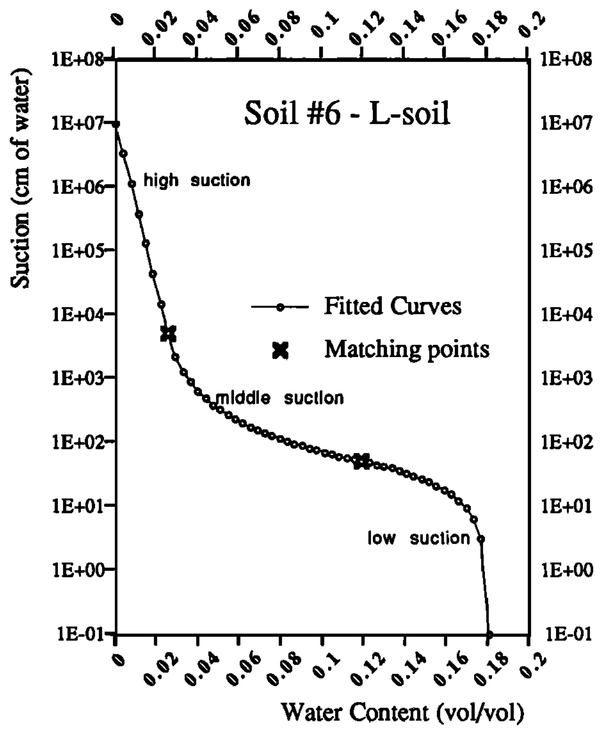

Figure 8. Continuously fitted retention curve for soil 6, showing the matching points where two different expressions join smoothly.

7. Capillary Drive Formula for New Expressions

The maximum capillary drive is defined in (1). On the basis of the results in Table 2 one does not need to consider the exponential part. Thus the expression for the capillary drive is

$$H_{cM} = \int_0^{h_{cm}} k_{rw}\, dh_c + \frac{h_{ce}}{\alpha - 1}\left[\left(\frac{h_{cm}}{h_{ce}}\right)^{-\alpha+1}\right] \qquad (32)$$

It remains to secure the explicit form of the first integral, namely,

$$\int_0^{h_{cm}} k_{rw}\, dh_c = \int_{x_m}^{1} -x^p \frac{dh_c}{dx}\, dx$$

$$= -h_{cm} x^{p+1}\left(\frac{A}{p+1} + \frac{Bx}{p+2} + \frac{Cx^2}{p+3}\right)\Bigg|_{x_m}^{1} \qquad (33)$$

where the parameters A, B, and C are defined as

$$A = -\frac{M}{x_m} + 2\,\frac{[(M+1)x_m - M]}{(1-x_m)^2} - a_{MS}x_m(2 + x_m) \qquad (34a)$$

$$B = 2\left\{a_{MS}(1 + 2x_m) - \frac{[(M+1)x_m - M]}{x_m(1-x_m)^2}\right\} \qquad (34b)$$

$$C = -3a_{MS} \qquad (34c)$$

Thus ultimately the expression for capillary drive is

$$H_{cM} = h_{cm}\left[x_m^{p+1}\left(\frac{A}{p+1} + \frac{Bx_m}{p+2} + \frac{Cx_m^2}{p+3}\right) - \left(\frac{A}{p+1} + \frac{B}{p+2} + \frac{C}{p+3}\right)\right] + \frac{h_{ce}}{\alpha - 1}\left[\left(\frac{h_{cm}}{h_{ce}}\right)^{-\alpha+1}\right] \qquad (35)$$

Table 4 provides a comparison between the values calculated using (35) and the values, $(H_{cM})_{\text{B-C}}$, obtained by using (7), which assumes the validity of the B-C power law to saturation and to zero suction. As a result, the B-C power law (without the gradual transition in the low-suction range) will overpredict the capillary drive. Some investigators [e.g., *White and Sully*, 1987; *Warrick and Broadridge*, 1992] have assumed that the appropriate capillary drive for the B-C expression is given by

Table 4. Various Estimations of the Capillary Drive and Comparison Between Them

Soil	H_{cM}	$(H_{cM})_{\text{B-C}}$	$H_{cM}/(H_{cM})_{\text{B-C}}$	$H_{cwf}/(H_{cM})_{\text{B-C}}$
1	73.15	117.73	0.6213	0.3112
2	63.75	92.02	0.6927	0.3774
3	63.70	95.94	0.6639	0.3048
4	150.80	246.45	0.6119	0.2996
5	75.24	103.93	0.7239	0.2285
6	23.37	32.17	0.7264	0.2548
7	210.15	343.29	0.6122	0.2999

Values are given in centimeters.

Table 3. Parameter Values and Goodness of Fit of New Expressions to the Seven Data Sets

	Soil Number						
	1	2	3	4	5	6	7
High suction	0.9658	0.9805	0.9835	0.9872	0.9105	0.8356	0.9183
Middle range	0.9842	0.9933	0.9945	0.9763	0.9947	0.9507	0.9568
Low suction	0.9078	0.9380	0.9734	0.8470	0.9169	0.9173	0.9640
HCE, cm	81.9	57.30	66.70	172.62	80.18	23.97	240.33
M	2.47	4.62	2.34	2.24	1.26	1.56	2.25
B-C λ	0.405	0.216	0.427	0.446	0.794	0.641	0.444
AMS	3.09	4.75	16.52	2.65	16.04	20.59	2.66
WCMS	0.3283	0.4539	0.2871	0.3494	0.2135	0.1187	0.3652
h_{cm}, cm	184.10	141.64	150.36	386.87	164.18	50.89	538.83

AMS is a_{MS}; WCMS is θ_m.

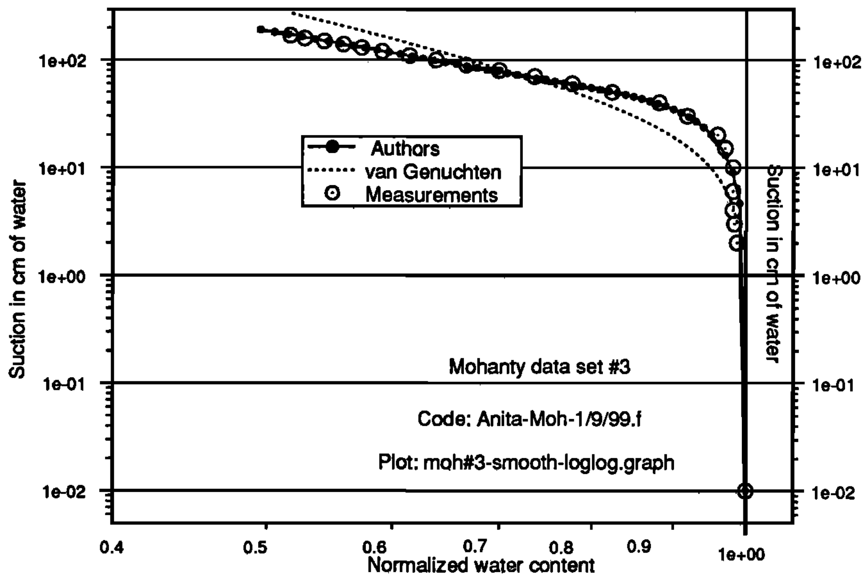

Figure 9. Fitted retention curve for Mohanty sample 3.

$$H_{cwf} = \int_{h_{ce}}^{\infty} k_{rw}\, dh_c = h_{ce}/(\alpha - 1) \qquad (36)$$

which integrates only the part of the retention curve outside the capillary fringe. Table 4 shows that (36) underestimates the capillary drive by about 50%.

8. Further Tests of Determination of Capillary Drive From Retention Data

In sections 5–7 a method was used to fit retention data to different expressions and it worked well. It was also suggested that values for relative permeabilities (and consequently for capillary drive) could not be secured reliably solely from the knowledge of the retention curve. That fact could not be verified on the seven data sets used as no associated relative conductivities were available. For this reason (at the instigation of one of the reviewers) another set of data of retention and conductivity, collected in an extensive field study [*Mohanty et al.*, 1997], was used. This set includes nine samples from a field site consisting of silty clay loam sediments. It should be noted that the surface horizons contain "visible root channels, worm holes and cracks" [*Mohanty et al.*, 1997, p. 2050]. Presumably, the set of data used in sections 6 and 7 did not include any form of "macropores."

Figure 9 displays the retention curve for sample 3 as fitted by the approach of this paper. The data are well fitted in that case. Figure 10 displays the retention curve for sample 9, the worst fit among the nine samples. For these nine samples the values of capillary drive were estimated as discussed in section 7, based strictly on the retention data, and are shown in Table 5. The estimates of capillary drives are in error by an order of

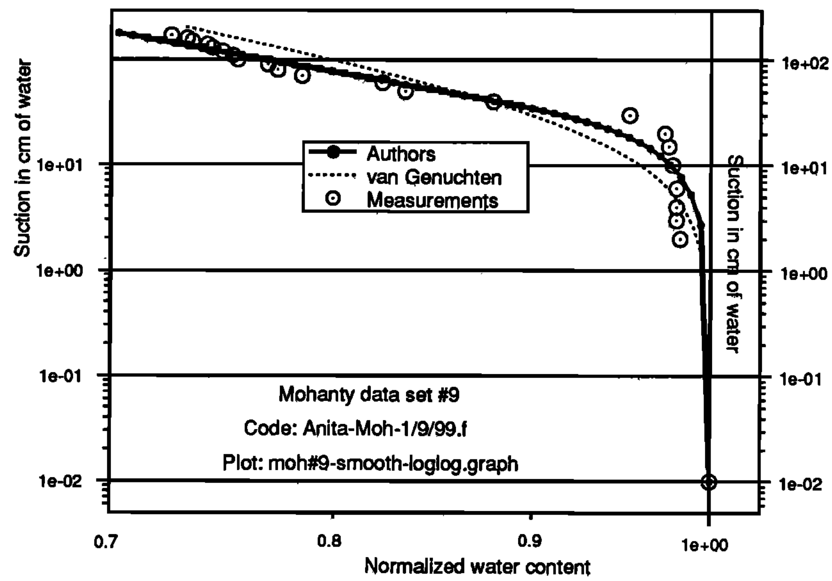

Figure 10. Fitted retention curve for Mohanty sample 9.

Table 5. Estimations of Capillary Drive by Different Approaches

Set	This Paper	Brooks-Corey	B-C Truncated	van Genuchten	Measurements
1	55.5774	76.0777	24.4065	2.4118	4.5009
2	32.9436	45.2042	13.1422	4.7678	3.5193
3	32.4097	44.3731	14.1298	3.1404	3.2451
4	61.5490	84.5176	23.8152	7.6386	3.5688
5	23.9819	31.5776	11.4966	1.1508	4.0189
6	36.0361	49.2584	16.6099	3.9043	4.6119
7	41.5028	56.8742	17.4735	1.5606	3.9095
8	14.6027	19.8439	7.7122	1.1663	3.0368
9	22.2607	29.2553	11.9239	0.7893	4.0542

Values are given in centimeters.

magnitude (except for the vG ones, even though the retention data were not particularly well fitted by the vG expression) as compared to the values derived from the measurements of conductivity as a function of suction.

These numbers support the view that retention data alone cannot lead to accurate estimates of conductivities, especially near saturation. Put differently, the curve fitting of data of retention and permeability must be conducted independently, especially near saturation. Figures 11 and 12 show the data of measured conductivities versus suction, with arithmetic scales for both axes, for samples 3 and 9, respectively, with a proposed new curve fitting of the data. The values of capillary drive given in the second column of Table 5 were obtained by extending the good B-C fit for retention for the middle range of suction to saturation, using the B-C relative permeability derived from the retention power law parameter, as provided by (5), as a function of normalized water content. That extension is not valid even if the retention curve itself is no longer described by the B-C power law but rather is described by the cubic expression of (26), which fits the data at low suction quite well at least for sample 3.

The data of conductivities, possibly because of macropores, drop extremely rapidly with suction, and as seen from Figures 11 and 12, the area under the curve of relative permeability versus suction is small, leading to the low values displayed in the last column of Table 5. We note that the curves of permeability versus suction are similar for samples 3 and 9 in spite of the disparity of the degree of fit for the retention curves of the two samples. That similarity was observed for the nine samples. What sort of expression would fit these data well? The choice of expression is not crucial provided that the value of the capillary drive, as obtained directly from the measurements, be preserved in the curve fitting. Following *Mohanty et al.* [1997] and for convenience, as such choice allows for an analytical formula for the capillary drive, the selected expression is

$$k_{rw} = e^{-(h_c - 0.01)/\Delta} \qquad h_c \le h_{ma} \tag{37a}$$

relative conductivity having been defined with respect to conductivity at a suction of 0.01 cm and h_{ma} being a suction where another expression is used, namely,

$$k_{rw} = e^{-(h_{ma} - 0.01)/\Delta}\left(\frac{h_{ma}}{h_c}\right)^{p/M} \qquad h_c \ge h_{ma} \tag{37b}$$

The parameters p and M are deduced from the retention curve. The requirements that (1) the two curves join smoothly at $h_c = h_{ma}$ and (2) the measured capillary drive, H_c^O, be preserved lead to values for the unknown parameters Δ and h_{ma}:

$$\Delta = \frac{H_c^O}{1 + \dfrac{M}{p - M}\, e^{-(p/M)}} \tag{38a}$$

$$h_{ma} = (p/M)\Delta \tag{38b}$$

It was verified that for the nine samples, (37b) described the data fairly well for $h_c \ge h_{cm}$. One could have tried a third type of expression in the range (h_{ma}, h_{cm}) to better describe that range. However, such refinement is probably not justified. For the estimation of infiltration capacity, a crucial quantity for the partition of a supply rate into runoff and infiltration, the essential parameter is the capillary drive, and the proposed expressions for relative conductivity versus suction provide it exactly from the observations. For the description of redistri-

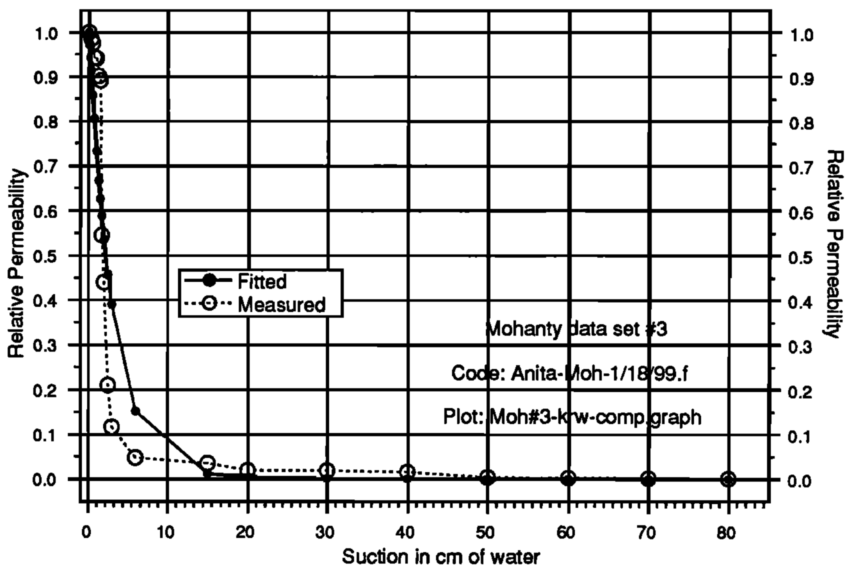

Figure 11. Fitted and measured values of relative permeability versus suction for Mohanty sample 3.

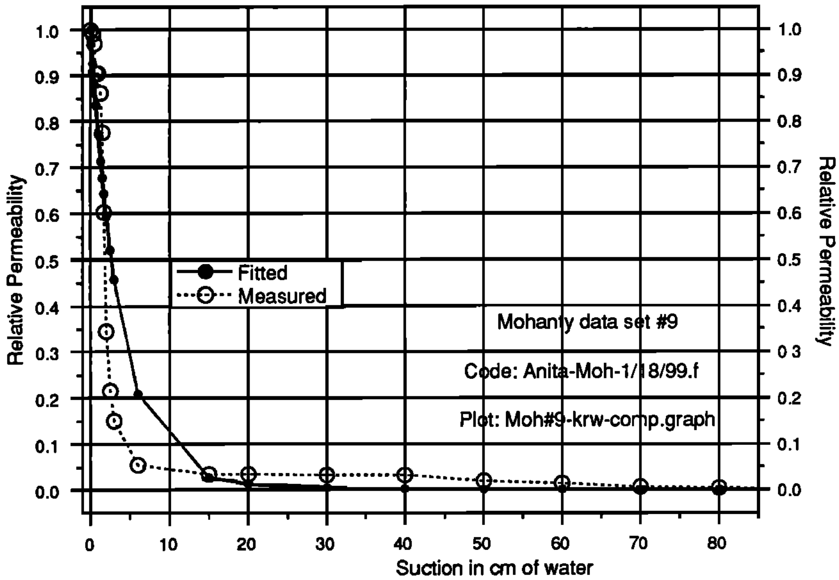

Figure 12. Fitted and measured values of relative permeability versus suction for Mohanty sample 9.

bution, evapotranspiration, and aquifer recharge the range of suctions involved are covered adequately by the B-C power law derived from the retention data and the exponential fit beyond.

9. Conclusions

If original data are available, a reliable procedure for data fit is to use the three expressions proposed here for high-, middle-, and low-suction ranges ((3), (9), and (26), respectively, with the understanding that normalized water content includes a nonzero value for the parameter serving as the B-C residual water content in the middle range). One can determine a value for capillary drive from the equations provided. Presuming that one has the correct value and only B-C parameters are available to describe the retention curve, it is recommended to extrapolate the B-C curve ((3), which is only valid for the middle range) by the exponential equation (9) for the high suctions. That procedure will not affect the estimate of the capillary drive. However, owing to the necessity of using the B-C expression in the capillary fringe and since the original data are not available, the estimate of the capillary drive using (7) will overestimate by 25 to 30%. Thus the calculated value should be discounted by roughly this amount.

Although the calculated value for capillary drive may not be exact, the data are well fitted in the low-suction range by the proposed expressions, and it is clear from Figures 1 and 2, for example, that many curves could pass through the scarce data points close to saturation. These curves can yield different values for capillary drive. Additionally, it was presumed that the relative permeability can be represented by the power law in the range of suction $0 \le h_c \le h_{ce}$ (the capillary fringe). However, A. Corey (personal communication, 1997) states that in the capillary fringe the relative permeability drops far less rapidly than when the vG expressions are used or when the proposed expressions in this research are used. However, the observed values of capillary drive when some macropores are involved (see Table 5) lead to a totally opposite conclusion. The root of the problem is the insistence on estimating a dynamic quantity on the basis of a static one. Unless experiments to measure infiltration rates as a function of time are performed, one cannot be sure that capillary drive values derived from experimental retention curves and "theoretical" relative permeabilities are reasonably accurate. Therefore one should perform such experiments and then fit the retention curves requiring a relation between the parameters so that the calculated capillary drive matches the experimental value derived from the infiltration test. Because capillary drive values are highly dependent on what happens near "natural saturation," they cannot be determined precisely from retention data and a "theoretical" relative permeability. Experimental determination of infiltration rates with time can provide a precise estimate of capillary drive, and we have shown that it is a practical matter then (as has been done with vG and B-C parameters by *Morel-Seytoux et al.* [1996]) to convert B-C parameters to modified R-N parameters and vice versa.

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