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# Physically Based Estimation of Soil Water Retention from Textural Data: General Framework, New Models, and Streamlined Existing Models

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Numerous models are in widespread use for the estimation of soil water retention from more easily measured textural data. Improved models are needed for better prediction and wider applicability. We developed a basic framework from which new and existing models can be derived to facilitate improvements. Starting from the assumption that every particle has a characteristic dimension $R$ associated uniquely with a matric pressure $\psi$ and that the form of the $\psi$–$R$ relation is the defining characteristic of each model, this framework leads to particular models by specification of geometric relationships between pores and particles. Typical assumptions are that particles are spheres, pores are cylinders with volume equal to the associated particle volume times the void ratio, and that the capillary inverse proportionality between radius and matric pressure is valid. Examples include fixed-pore-shape and fixed-pore-length models. We also developed alternative versions of the model of Arya and Paris that eliminate its interval-size dependence and other problems. The alternative models are calculable by direct application of algebraic formulas rather than manipulation of data tables and intermediate results, and they easily combine with other models (e.g., incorporating structural effects) that are formulated on a continuous basis. Additionally, we developed a family of models based on the same pore geometry as the widely used unsaturated hydraulic conductivity model of Mualem. Predictions of measurements for different suitable media show that some of the models provide consistently good results and can be chosen based on ease of calculations and other factors.

ABBREVIATIONS: AD, Arya–Dierolf; AP, Arya–Paris; CNEAP, continuous near-equivalent of the AP model; PC, proportional cylinder; UCNEAP, uniform-mass-distribution version of the continuous near-equivalent of the AP model.

The need to quantify unsaturated hydraulic properties for predicting subsurface transport phenomena, coupled with the difficulty of measuring these properties, has motivated efforts to develop property-transfer models that estimate the hydraulic characteristics based on more easily measured properties such as soil texture. Such models are usually classified as either physically or empirically based (Haverkamp et al., 2002), although many models are not purely one or the other.

In this paper we focus on computing water retention from particle-size distribution, rather than unsaturated hydraulic conductivity from water retention (Mualem, 1976) or multiple hydraulic properties from various inputs (Schaap et al., 2001). We emphasize models that have some type of physical basis as opposed to empirical relationships formulated using a database. Many different models have been developed for this purpose (e.g., Arya and Paris, 1981; Haverkamp and Parlange, 1986; Mishra et al., 1989; Chan and Govindaraju, 2004; Winfield, 2005). Models formulated to apply to particular locations or types of media, as long as they are applied to these cases, usually produce the most accurate results. Models of wider applicability involve compromises that are likely to make them less accurate in general. Different models have different virtues and defects, for example, performing better or worse in certain parts of the moisture range. Particle-size–based models need further development to apply to soils with complex structure. Thus, it is to be expected that new models continually need to be developed, either to apply to a specific site or material or to produce results of improved reliability or applicability.

Because closely related concepts are involved in many of the published property transfer models, it is possible, by recognizing common features, to create a general theoretical framework that ties together a family of individual models. General models of this type for unsaturated hydraulic conductivity are presented in papers such as those of Mualem and Dagan (1978), Hoffmann-Riem et al. (1999), and Kosugi (1999). Within a general framework, explicit specification of a set of conditions designates a particular model. This situation is analogous to the way a general solution of a differential equation encompasses a wide range of particular solutions, any of which can be singled out by specifying conditions. In this way a general model can serve as a template for generating new models that may be superior in basic reliability of predictions or in accuracy of results for particular situations. A general theoretical framework is also useful for showing how different models within a family are related, for identifying essential and optional assumptions, and for facili-

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tating conversion of results or applications from one model to another.

In this paper we derive a general framework for property transfer models and illustrate its application with different assumptions that lead to various particular models, both new and previously published. The emphasis here is on the intermediate and wet range of the retention curve, characterized by the capillary emptying and filling of individual pores at particular values of $\psi$, as opposed to the drier range characterized by water in films. The general framework is intended to apply to either drying or wetting curves, although hysteresis is not explicitly treated. In this category of models that are based on hypothesized pore-scale physical relationships, the Arya and Paris (1981) (AP) model has dominated unsaturated-zone applications (e.g., Buczko and Gerke, 2005; Vaz et al., 2005) despite shortcomings that have been known since its introduction (Haverkamp and Parlange, 1982). Many modifications and adaptations of the AP model have been developed (e.g., Basile and D'Urso, 1997). Much of this paper pertains in particular to the AP model and models that are related to it or have a similar purpose. Another objective is to produce a model that gives results essentially identical to those of the AP model but is formulated on a continuous basis and does not depend on the choice of interval size for particle-size data.

## General Framework

The following are four basic assumptions for a property-transfer model that goes from pore-size distribution to retention:

1. Every particle has a characteristic dimension $R$.
2. Every $R$ value is uniquely associated with a matric pressure $\psi$.
3. Every $R$ value is uniquely associated with a local void ratio $e_l$ or, equivalently, a local porosity $\phi_l$, with $e_l = \phi_l/(1 - \phi_l)$.
4. At a given volumetric water content, $\theta$, filled pores are those in which $\psi$ is less than or equal to a certain value associated with that $\theta$.

These assumptions lead to a general model in which $\theta$ at a given $\psi$ equals the integration of all pores filled at that $\psi$:

$$\theta(\psi) = \int_0^{R(\psi)} e_l(R)F(R)\mathrm{d}R \qquad [1]$$

where $F$ is the volumetric distribution of particle size (normalized to $1 - \phi$, the fraction of the total sample space occupied by particles, where $\phi$ is the porosity of the medium as a whole). Particular models can be generated by assigning functional forms to $R(\psi)$ and $e_l(R)$. The form of $R(\psi)$ can be an arbitrary choice, such as a straight line or an exponential curve on a graph of $R$ versus $\psi$. Typically, one would choose a relation constrained by physical concepts, for example, that $\psi$ relates to pore size by capillarity and that pore size relates to $R$ by a hypothetical geometric construction. In effect, for each particle there is an associated pore, the pore size being specified as a function of the particle size, and for each pore size there is a characteristic $\psi$ value at which the pore fills or empties.

The function $e_l(R)$ could be chosen to represent a systematic variation of packing efficiency with particle size. For example, an increase of $e_l$ in the clay size range might express the tendency for large (e.g., interaggregate) pores to be more prevalent in fine-textured soils. In most models, however, and for the rest of this paper a fifth assumption is adopted:

5. Void ratio is has a uniform value, symbolized $e$, throughout the medium.

This assumption is valid to the extent that the structure of the medium is uniform in a continuum sense, subject to the validity of a representative elementary volume smaller than the scale at which hydraulic processes are to be considered. Assumption 5 simplifies the general model to

$$\theta(\psi) = e\int_0^{R(\psi)} F\left(R(\psi)\right)\mathrm{d}R = eF_{\mathrm{cum}}\left(R(\psi)\right) \qquad [2]$$

where $F_{\mathrm{cum}}$ is the cumulative volumetric distribution of particle size (normalized to $\phi$). This relation between the functions representing water retention and particle-size distribution is probably what suggests the term *shape similarity* in many discussions of property transfer models. In general this term is only loosely applicable because $R(\psi)$ can be nonlinear.

A common but nonessential assumption is:

6. Particle density $\rho_p$ is the same for all particles.

Assumption 6 is not valid in general because for natural media, the material composition of particles can be expected to vary with size (e.g., clay minerals predominate at the smallest sizes). Because particle-size distributions are usually reported on a mass basis, however, this assumption is also used here. It permits the substitution for $F_{\mathrm{cum}}(R)$ of the mass particle-size distribution:

$$M_{\mathrm{cum}}(R) = \frac{\rho_p}{\rho_p}F_{\mathrm{cum}}(R) = \frac{e}{\phi}F_{\mathrm{cum}}(R) \qquad [3]$$

where $\rho_b$ is the bulk density and $M_{\mathrm{cum}}$ is the cumulative mass distribution of particle size (normalized to 1). In terms of $M_{\mathrm{cum}}$, the general model can be expressed as

$$\theta(\psi) = \phi M_{\mathrm{cum}}\left(R(\psi)\right) \qquad [4]$$

An important subset of the possible models defined by Eq. [2] or [4] uses additional geometric assumptions:

7. Particles are spheres with effective radius $R$.
8. Pores are cylinders with effective radius $r$ and length $h$.

For unsaturated pore-water relations, there one more usual assumption:

9. Capillary behavior determines whether a pore is empty or full, that is, $r = C_c/\psi$, where $C_c$ is a parameter that depends on surface tension and contact angles, and equals about 130 μm-kPa for typical soil water.

Setting the void ratio equal to the volume of a cylindrical pore divided by the volume of a spherical particle relates the three geometric parameters $R$, $r$, and $h$:

$$r = \left(\frac{4R^3e}{3h}\right)^{1/2} \qquad [5]$$

## Development of Particular Models

To generate a particular model, the necessary steps are (i) choose a geometric relation that defines $h$ in terms of $R$ or $r$; (ii) use the chosen relation to eliminate h from Eq. [5] above, thus producing a R($\psi$) function that characterizes the model; and (iii) put the $R(\psi)$ formula into the general Eq. [4] (or Eq. [2]) to produce the particular property transfer model, $\theta(\psi)$ equal to a function of $M(R)$.

### Proportional Cylinder (PC) Model

A simple model is based on the assumption of a fixed cylindrical shape for each pore, length proportional to radius:

$$h = C_{\mathrm{PC}} r \qquad [6]$$

where $C_{\mathrm{PC}}$ is the proportionality constant. This is the pore-shape assumption used by Mualem (1976, Fig. 1) in his capillary-bundle model for hydraulic conductivity. Applying the above capillary and geometric relations,

$$\psi = C_{\mathrm{c}} \left[ \frac{3C_{\mathrm{PC}} \left( \frac{C_{\mathrm{c}}}{\psi} \right)}{4R^3 e} \right]^{1/2} \qquad [7]$$

which reduces to

$$R = \frac{C_{\mathrm{c}}}{\psi} \left( \frac{3C_{\mathrm{PC}}}{4e} \right)^{1/3} \qquad [8]$$

In Eq. [4] this relation gives the retention curve:

$$\theta(\psi) = \phi M_{\mathrm{cum}} \left( \frac{C_{\mathrm{c}}}{\psi} \left[ \frac{3C_{\mathrm{PC}}}{4e} \right]^{1/3} \right) \qquad [9]$$

This equation defines a family of proportional cylinder (PC) models that differ in the value given to $C_{\mathrm{PC}}$.

### Arya–Dierolf Model

Arya and Dierolf (1992) (AD) created a model with the assumption that all pores have equal length, $h(r) = h_{\mathrm{AD}}$ for all $r$. Pores corresponding to the smaller particles then have extremely small $r$ in order for their volume to take the value necessitated by the particle size and void ratio. Following the steps as above for substitution into Eq. [4], the AD model is

$$\theta(\psi) = \phi M_{\mathrm{cum}} \left( \left[ \frac{C_{\mathrm{c}}}{\psi} \right]^{2/3} \left[ \frac{3h_{\mathrm{AD}}}{4e} \right]^{1/3} \right) \qquad [10]$$

As before, the argument of the function $M_{\mathrm{cum}}(R)$ is the formula for $R(\psi)$ that defines the model. In tests with five soils, Arya and Dierolf (1992) found that $h_{\mathrm{AD}}$ values between 3 and 15 mm produced good predictions and that a value of 10 mm should work well for many soils. A virtue of this model is that sensitivity to the value of $h_{\mathrm{AD}}$ is low.

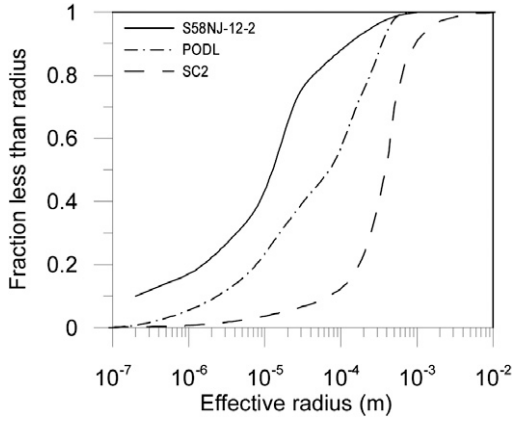


FIG. 1. Cumulative particle-size distributions of the three test media.

### Analogous Formulation of the AP Model

Arya and Paris (1981) assumed a complicated geometric relation in which $r$ is specified by a formula that varies with $M(R)$ and that has a parameter whose value is determined empirically from a database of properties for various soils. This model is formulated in finite intervals, most applications having about 5 to 20 intervals for a 0- to 1-mm range of $R$. Haverkamp and Parlange (1982) and others have criticized the AP model for its interval-size dependence on the grounds that a pore radius would not fundamentally depend on arbitrary choices made in analyzing samples. Arya and Paris (1982) responded that the discrepancy this causes is acceptable because the variation in choice of intervals makes only a small difference. It is clearly desirable, however, to formulate a near-equivalent of the AP model that is based on continuous functions and therefore avoids interval-dependence completely. Because the AP model cannot be formulated exactly using the continuous formulas of our general framework, we first reformulate it in an analogous finite-interval framework before developing a near-equivalent model that fits the continuous-function framework.

The AP model matches $\psi_i$ at the center of each interval with an average water content specified by an analog to the continuous-$\theta$ Eq. [4]:

$$\bar{\theta}_i = \frac{\phi}{2} \left[ M_{\mathrm{cum}}(R_{\mathrm{b}i-1}) + M_{\mathrm{cum}}(R_{\mathrm{b}i}) \right] \qquad [11]$$

where subscript $i$ indicates the midpoint and subscript b$i$ indicates the right-side bound of interval i. A discrete-interval version of the basic cylindrical-pore geometric relation, Eq. [5], is

$$r_i = \left( \frac{4R_i^3 e}{3h_{\mathrm{avg}\text{-}i}} \right)^{1/2} \qquad [12]$$

where $h_{\mathrm{avg}\text{-}i}$ is the length of a single average pore for interval $i$, with the total number of such pores taken to equal the number of particles in the interval. Early in Arya and Paris's development, they provisionally specified the length $h_{\mathrm{avg}\text{-}i}$ to equal the particle diameter $2R_i$, which with the capillarity relation gives

$$R_i = \frac{C_{\mathrm{c}}}{\psi_i} \left( \frac{3}{2e} \right)^{1/2} \qquad [13]$$

This equation is equivalent to the PC model if $C_{\mathrm{PC}}$ is given the value $(6/e)^{1/2}$. The AP model departs from the PC model in making $h_{\mathrm{avg}\text{-}i}$ greater than $2R_i$ by a factor depending on a parameter $\alpha$ and the number of particles in the interval:

$$h_{\text{avg-}i} = 2R_i(m_o n_i)^{(\alpha-1)} \quad [14]$$

where $m_o$ is a standard mass, taken to equal 1 g in AP formulas, and $n_i$ is the number of particles, per total sample mass, in interval $i$. The formulas published by AP do not show it explicitly, but inclusion of $m_o$ is essential for dimensional consistency and to avoid sample-size dependence of the results. The value of 1 g for this standard mass is implicit in the example calculations of Arya and Paris (1982). Substituting this enhanced pore length, Eq. [14], into the geometric relation, Eq. [12], leads to

$$R_i = \frac{C_c}{\psi_i}\left[\frac{3(m_o n_i)^{(\alpha-1)}}{2e}\right]^{1/2} \quad [15]$$

Because the original version of the AP model uses the number distribution $n_i$ instead of the mass distribution $M(R)$, it is useful to relate these quantities to each other. For this purpose, consider $n(R)$, defined such that $n(R)\mathrm{d}R$ is the number of particles in the infinitesimal interval $R$ to $R + \mathrm{d}R$, per unit sample mass. Because $M(R)\mathrm{d}R$ is the mass of particles in that interval, per unit sample mass, $M(r)$ is equal to $n(R)$ times the mass of a single particle of radius $R$. This gives

$$n(\mathrm{R}) = \frac{3M(\mathrm{R})}{4\pi R^3 \rho_p} \quad [16]$$

In the finite interval $i$, the number of particles per total sample mass is

$$n_i = \int_{R_{bi-1}}^{R_{bi}} n(\mathrm{R})\mathrm{dR} = \frac{3}{4\pi\rho_p}\int_{R_{bi-1}}^{R_{bi}} \frac{M(\mathrm{R})}{R^3}\mathrm{dR} \quad [17]$$

which in combination with Eq. [15] gives

$$\psi_i = \frac{C_c}{R_i}\left\{\frac{3}{2e}\left[\frac{3m_o}{4\pi\rho_p}\int_{R_{bi-1}}^{R_{bi}} \frac{M(\mathrm{R})}{R^3}\mathrm{dR}\right]^{\alpha-1}\right\}^{1/2} \quad [18]$$

After inverting for $R_i(\psi_i)$ this equation becomes an analog to the $R(\psi)$ function needed to specify a particular model in the general framework. Pairing $\psi_i$ with $\theta_i$ from Eq. [11] gives AP model results. In cases where $M(R)$ can be expressed as an algebraic form that permits Eq. [18] to be analytically inverted, the resulting $R_i(\psi_i)$ function could be substituted in Eq. [11] to give a single-equation expression of the AP model.

## Continuous Near-Equivalent of the AP Model

For a continuous-function equivalent of the AP model, consider the limit as the particle-size interval size goes to zero ($R_{bi-1} \rightarrow R_i$ and $R_{bi} \rightarrow R_i$). In this limit,

$$\begin{aligned} \bar{\theta}_i &\rightarrow \theta \\ \psi_i &\rightarrow \psi \\ R_i &\rightarrow R \\ n_i &\rightarrow n\mathrm{dR} \end{aligned} \quad [19]$$

and the number of particles (within a sample of size $m_o$) is

$$m_o n(\mathrm{R})\mathrm{dR} = \frac{3m_o}{4\pi\rho_p}\frac{M(\mathrm{R})}{R^3}\mathrm{dR} \quad [20]$$

The equivalent of Eq. [18] is

$$\psi = \frac{C_c}{R}\left[\frac{3}{2e}\left(\frac{3m_o}{4\pi\rho_p}\frac{M(\mathrm{R})\mathrm{dR}}{R^3}\right)^{\alpha-1}\right]^{1/2} \quad [21]$$

This formula explicitly shows the inherent AP interval-size dependence. As $\mathrm{d}R \rightarrow 0$, if $\alpha > 1$, then $\psi \rightarrow 0$, and if $\alpha < 1$, then $\psi \rightarrow \infty$. Thus, a continuous form of the original AP model is impossible unless the value chosen for $\alpha$ is 1. That choice makes this model identical to the proportional cylinder model, and Eq. [21] becomes equivalent to Eq. [7] ,with $C_{PC} = (6/e)^{1/2}$. For the typical case of $\alpha$ valued between 1 and 2, $\psi$ goes to zero more slowly than $\mathrm{d}R$ does, which supports the claim that AP modeled retention curves are not very sensitive to interval size. The sensitivity to interval size increases for $\alpha$ significantly greater than 1.

A continuous-function formula that closely approximates the AP model can be obtained by replacing the differential $\mathrm{d}R$ in Eq. [21] with a finite expression $\delta R$, selected for consistency with earlier practice concerning interval size. Arya and Paris (1981) tested the model with data sets having five to eight intervals but noted that "a more detailed fractionation of the soil would be desirable" (p. 1030). To demonstrate the model calculations, Arya and Paris (1982) chose the case of 13 intervals over the $R$ range from 0 to 1000 μm, whose bounds are listed in Table 1. Considering this case as an effective standard, the average value of the ratio of interval width to interval midpoint is 0.7154. This value is close to the value of two-thirds that would result from defining $\delta R$ to equal the interval's lower bound. Approximate consistency with previous AP results can be achieved by replacing the differential $\mathrm{d}R$ in [21] with $\delta R = (2/3)R$, giving

$$\psi = \frac{C_c}{R}\left\{\frac{3}{2e}\left[\frac{m_o}{2\pi\rho_p}\frac{M(\mathrm{R})}{R^2}\right]^{\alpha-1}\right\}^{1/2} \quad [22]$$

This equation is a continuous near-equivalent of the AP model (CNEAP). Practical use of CNEAP, for given $M_{cum}(R)$ and $\phi$, would usually be as follows: (i) compute $M(R)$ from $M_{cum}(R)$ by numerical differentiation (e.g., Ralston, 1965) or by fitting an analytical function to the data and differentiating that function;

TABLE 1. Particle-size intervals used by Arya and Paris (1982) for the soil S58NJ-12-2. Radii are in micrometers.

| $R$ (upper bound) | $R_{average}$ | $\Delta R$ | $\Delta R/R_{average}$ |
|---|---|---|---|
| 0 | | | |
| 0.5 | 0.25 | 0.5 | 2.0000 |
| 1 | 0.75 | 0.5 | 0.6667 |
| 2.5 | 1.75 | 1.5 | 0.8571 |
| 5 | 3.75 | 2.5 | 0.6667 |
| 10 | 7.5 | 5 | 0.6667 |
| 15 | 12.5 | 5 | 0.4000 |
| 25 | 20 | 10 | 0.5000 |
| 50 | 37.5 | 25 | 0.6667 |
| 100 | 75 | 50 | 0.6667 |
| 250 | 175 | 150 | 0.8571 |
| 350 | 300 | 100 | 0.3333 |
| 500 | 425 | 150 | 0.3529 |
| 1000 | 750 | 500 | 0.6667 |

TABLE 2. Media for illustrative model tests.

| Sample | Location | Sampled material | Depth | Textural class | Porosity | Reference |
|---|---|---|---|---|---|---|
| S58NJ-12-2 | Middlesex County, NJ | B23t horizon | 0.6 m | loam | 0.362 | Soil Conservation Service–USDA (1974) |
| PODL-1-6.6 | Morgan Creek watershed, MD | Coastal-plain sediments | 6.6 m | sandy loam | 0.3647 | Perkins (personal communication, 2004) |
| SC2 | Sheep Creek, Mojave Desert, CA | Fluvial deposit | 0.7 m | sand | 0.397 | Winfield et al. (2006) |

and (ii) for each value of $R$ for which a $\theta(\psi)$ point is desired, compute $\psi$ from Eq. [22] and $\theta$ from Eq. [4]. Alternatively, if a directly usable $\theta(\psi)$ function is desired, one can use an $M(R)$ function that permits inversion. The inverted Eq. [22] substituted into Eq. [4] gives a single-equation $\theta(\psi)$ formula.

Additional simplification is possible by assuming a particular universal $M(R)$ function for the purpose of relating $\psi$ and $R$. For this purpose we have tested a simple functional form, namely, a uniform mass distribution of particles up to the maximum particle size $R_{max}$:

$$M(R) = 1/R_{max} \text{ for } R \leq R_{max} \quad [23]$$

$$M(R) = 0 \text{ for } R > R_{max} \quad [24]$$

This function is used only for the purpose of relating $\psi$ and $R$, not for the determination of the retention curve with Eq. [4], so the effect on model results is small. Substitution into the continuous formula Eq. [22] gives an $R(\psi)$ for use in Eq. [4] to yield a uniform-mass-distribution version of the continuous near-equivalent of the AP model (UCNEAP):

$$\theta(\psi) = \phi M_{cum}\left(\left[\frac{C_c}{\psi}\right]^{1/\alpha}\left[\frac{3}{2e}\right]^{1/2\alpha}\left[\frac{m_o}{2\pi\rho_p R_{max}}\right]^{\frac{\alpha-1}{2\alpha}}\right) \quad [25]$$

This is a single-equation continuous-function model that approximates the AP model.

## Model Tests

### Data

We chose data sets for three samples to test and illustrate model results (Table 2). The data were for soils and sediments with relatively simple structure that are expected to be suitable for the class of models addressed here, but spanning a significant textural range. Sample S58NJ-12-2, the same chosen as an illustrative example by Arya and Paris (1982), is relatively coarse, from a shallow depth, and was repacked before measurement of $\theta(\psi)$, so features of natural soil structure were destroyed.

TABLE 3. Coefficient of determination $R^2$ for model fits to data.

| Model† | S58NJ-12–2 | PODL | SC2 |
|---|---|---|---|
| PC $C_{PC}$ = 3 | 0.6875 | 0.9385 | 0.5329 |
| PC $C_{PC}$ = 100 | 0.8275 | 0.9323 | 0.9777 |
| PC $C_{PC}$ = 1000 | 0.9222 | 0.8899 | 0.9429 |
| PC $C_{PC}$ = 10000 | 0.9823 | 0.8006 | 0.8268 |
| AD $h_{AD}$ = 1 cm | 0.9849 | 0.9475 | 0.9911 |
| AP-13 int | 0.9789 | 0.9518 | 0.9960 |
| CNEAP | 0.9836 | 0.9541 | 0.9848 |
| UCNEAP | 0.9657 | 0.9553 | 0.9807 |

† PC, proportional cylinder; AD, Arya–Dierolf; AP, Arya–Paris; CNEAP, continuous near-equivalent of the AP model; UCNEAP, uniform-mass-distribution version of the continuous near-equivalent of the AP model.

Samples PODL-1-6.6 and SC2 are coarser in texture and were measured as minimally disturbed cores, although their structural complexity is limited because of coarseness (especially SC2) and origination from depths below the surface region of most intense weathering (especially PODL-1-6.6). Figure 1 shows the particle-size distributions. Table 3 gives the coefficient of determination, as a measure of goodness-of-fit, for eight models fit to the three samples.

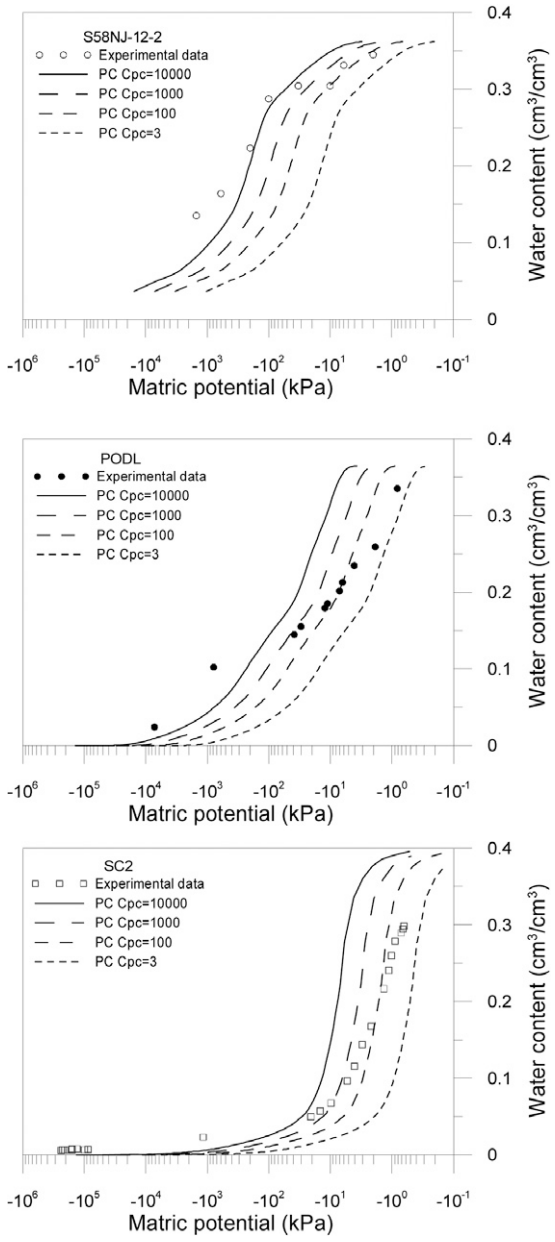


FIG. 2. Proportional cylinder (PC) model with four values of the proportionality constant ($C_{PC}$) applied to the three test media: S58NJ-12-2, PODL-1-6.6, and SC2.

## Results

Predictions of the PC model for four values of the parameter $C_{PC}$ are given in Fig. 2. One might expect that in a sandy medium, the length of a pore associated with a particular particle would be on the order of the particle radius, implying a small value of $C_{PC}$. The $(6/e)^{1/2}$ value suggested by comparison to other models equals about 3 for typical media. The optimal value of $C_{PC}$, however, is about 100 or greater for all three test media. A $C_{PC}$ value of 3 seriously underpredicts each case. Even with larger $C_{PC}$ values, the model has a bias toward underprediction in the dry range.

The AD model produces better predictions than the PC model for any of the illustrated $C_{PC}$ values (Fig. 3). The $R^2$ values in Table 3 show that if one selects the optimal $C_{PC}$ value, the PC model can fit nearly as well as the AD model, but the AD model has been applied here with a standard $h_{AD}$ equal to 10 mm and no further parameter optimization. With its enhanced contribution of the smallest-radius pores, the AD model does not underpredict the low θ range as badly as the PC model does. The $h_{AD}$ value of 10 mm works well for S58NJ-12-2 and SC2 and mostly overpredicts θ for PODL-1-6.6. In all three cases, it gives a fit close to that of the original AP model.

The CNEAP model (Eq. [22]) gives results nearly identical to those of the original AP model with 13 intervals, as shown in Fig. 4. Results of the UCNEAP (Eq. [24]) differ slightly from the AP model. Although better agreement may be possible with a different $M(R)$ dependence embedded in the $R(\psi)$ relation, it is not necessary to have such a dependence in the first place. In practice the simpler UCNEAP model may generally perform as well as the AP model.

## Discussion

From the starting point that a property transfer model for predicting water retention curves from particle-size distributions can be characterized by a particular relationship between particle size and matric pressure, we evaluated several different models within a unifying framework. The applications developed here fall within a particular subset of the possibilities, and all of them consider particles as spheres and pores as cylinders. It is worthwhile to explore wider possibilities, especially models that are not tied to the sphere–cylinder geometry and models that account for hysteresis. It also is not necessary to base the particular model on a geometric form; an algebraic $R(\psi)$ could be selected for use in Eq. [4] based on fundamental or empirical reasoning or mathematical convenience. The models considered are applicable for media of simple structure, although modification could allow them to represent properties of media with significant structural features such as aggregation and macropores (e.g., Nimmo, 1997).

Two models explored here, the PC and AD models, are especially simple. The PC model may have advantages in combining with the Mualem (1976) model for unsaturated hydraulic conductivity, as the combination would use the same hypothetical geometry for both water retention and unsaturated $K$ property transfer. The AD model is as simple to apply as the PC model and probably gives more realistic predictions. Both of these models seem only to work well if it is assumed that cylindrical pores have length much greater than their diameters, for example, about 10 mm for a 10-μm radius. This may reflect a departure of the physical processes of water retention from the cylinder-model assumptions, especially in the low θ range where adsorption leads to water retention in both small and large pores independent of capillary filling. Cylinder elongation in the model can artificially compensate for this effect. Such compensation, however, seems to be needed not only for pores sized at the lower limits of capillarity but also for ψ values up to about −10 kPa, corresponding to capillary radii of tens of microns. These considerations do not preclude practical use of such models, but it should be recognized that the hypothetical elongated cylinders probably do not approximate actual pore shapes.

With additional assumptions that relate the finite intervals and the $M(R)$ dependence of $R$–ψ to the types of data and calculations the AP model was designed for, we created the CNEAP model within the general framework. This new model produces results consistent with earlier AP model calculations but uses a formula that is a continuous function. An additional simplification produces the UCNEAP model, which allows single-function representation of θ(ψ) closely resembling the AP model.

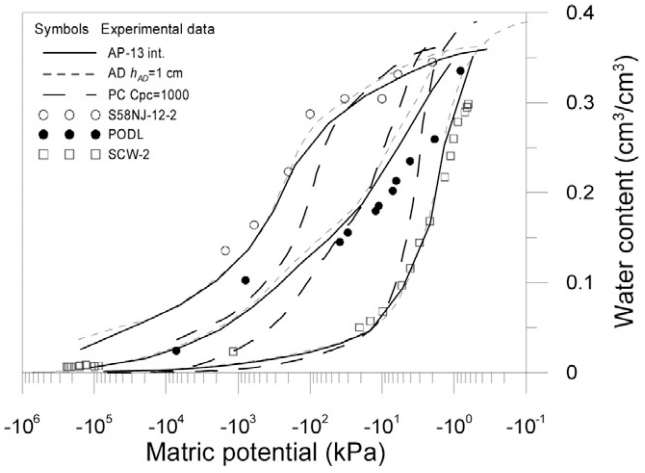


FIG. 3. The Arya–Dierolf (AD) model with $h_{AD}$ = 10 mm compared with the proportional cylinder (PC) model with $C_{PC}$ = 1000 and the original Arya–Paris (AP) model applied to the three test media.

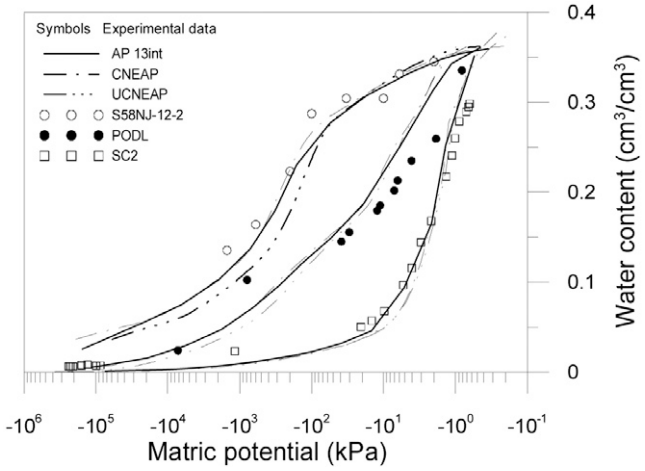


FIG. 4. The Arya–Paris (AP) model with 13 intervals, the continuous near-equivalent of the AP (CNEAP) model, and the uniform-mass-distribution version of the continuous near-equivalent of the AP (UCNEAP) model, all with α = 1.38, applied to the three test media.

The deviation from 13-interval AP-model results of the CNEAP, UCNEAP, and AD models (Fig. 4) is probably less than the deviations within the AP model itself when different interval sizes are used. The versions examined here are calculable by directly applying an algebraic formula rather than manipulating tables of data and intermediate results. The AD and UCNEAP models also eliminate the dependence of the geometric pore-cylinder relation on the particle-size distribution. The CNEAP model is equivalent to AP except for the finite-versus-continuous distinction and so should produce essentially identical results for diverse media and circumstances.

The continuous form of the alternative models has value beyond its assurance that interval size cannot affect results. In developing new models (e.g., incorporating structural effects), when the development involves combining AP with one or more other models, and those other models are formulated on a continuous basis, the combining is simpler and more straightforward if the AP is also on a continuous basis. Such combinations have been shown to be fruitful, for example, in the models of Mishra et al. (1989), Nimmo (1997), and Hwang and Powers (2003). A further advantage is that the continuous AP version, expressible in two equations, can be simpler than the original discrete-interval version. It also avoids the discontinuity of slope in the modeled curve that occurs where interval size undergoes a significant change.

Certain cases of the different models are exactly equivalent. The AP, CNEAP (Eq. [22]), and UCNEAP (Eq. [24]) models are equivalent to the PC model if we set $\alpha = 1$ and $C_{PC} = (6/e)^{1/2} \approx 3$. The UCNEAP formula is very similar to the AD formula, as may be seen by writing Eq. [25] as

$$\theta(\psi) = \phi M_{cum}\left(\frac{C_U}{\psi^{1/\alpha}}\right) \qquad [26]$$

and Eq. [10] as

$$\theta(\psi) = \phi M_{cum}\left(\frac{C_{AD}}{\psi^{2/3}}\right) \qquad [27]$$

where $C_U$ and $C_{AD}$ lump together various constants that appear in the UCNEAP and AD formulas. If we set $\alpha$ in Eq. [26] to 1.5 (close to the original AP recommendation of 1.38), then equating $C_U$ and $C_{AD}$ gives the condition for equivalence of the two models, namely, that the product of $h_{AD}$ and $R_{max}^{1/2}$ equals 15.5 $mm^{3/2}$. Thus the UCNEAP model with $\alpha = 1.5$ is equivalent to the AD model with the traditional $R_{max}$ value of 1 mm and an $h_{AD}$ value of 15.5 mm, which falls essentially within the range that Arya and Dierolf (1992) found to work well. Because the UCNEAP model approximates the results of the AP model, this equivalence illustrates a close connection between the AD and AP models.

The fits in Fig. 3 and 4 and the coefficients of determination in Table 3 show that for each of the three test media, the AD, AP, CNEAP, and UCNEAP models yield predictions of about equal quality. All of the model fits are less good for PODL than for the other two samples. Water retention in the PODL sample seems to depend, more than for the other two media, on non–texture-related structural aspects of the pore space. Not surprisingly, the underlying assumptions of the general pore-size–particle-size framework do not apply equally well to all soils.

## Conclusions

The generalized framework represented by Eq. [4] for property transfer from particle-size distribution to water retention relies on few assumptions and makes explicit the meaning of shape similarity in relating particle-size distributions to water retention curves. It provides a basis for straightforward incorporation of assumptions, such as those that specify pore-particle geometry, to derive property-transfer models for fundamental investigations or practical use. Such new models may have greater simplicity, reliability, or applicability than existing models. Recasting particular models within this approach clarifies relationships among them, for example that the PC (fixed pore-shape) model is a special case of the AP model, and that the AD model under certain conditions is essentially equivalent to the AP model. The framework also highlights the assumptions, which vary in realism and necessity, that go into these models.

Using this framework produces an explicit algebraic expression (Eq. [18] and [11]) of the AP model (Ayra and Paris, 1981) and various related models. Nearly equivalent to AP are the CNEAP (Eq. [22] and [4]) and UCNEAP (Eq. [25]) models, as well as the AD model (Arya and Dierolf, 1992) (Eq. [10]). These models give results closely approximating those of the original AP model and facilitate applications in which it is desirable to have consistency with the large body of previous Arya–Paris modeled results. These alternative models are formulated on a continuous basis, and two of the models (AD and UCNEAP) are represented by single-formula mathematical functions. These features expand their versatility for algorithmic implementation and enhance opportunities for combining, extending, adapting, and refining them for new and traditional uses.


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## References


Arya, L.M., and J.F. Paris. 1981. A physicoempirical model to predict the soil moisture characteristic from particle-size distribution and bulk density data. Soil Sci. Soc. Am. J. 45:1023–1030.

Arya, L.M., and J.F. Paris. 1982. Reply to "Comments on a physicoempirical model to predict the soil moisture characteristic from particle-size distribution and bulk density data. Soil Sci. Soc. Am. J. 46:1348–1349.

Arya, L.M., and T.S. Dierolf. 1992. Predicting soil moisture characteristics from particle-size distributions: An improved method to calculate pore radii from particle radii. p. 115–124. *In* M.Th. van Genuchten, F.J. Leij, and L.J. Lund (ed.) Indirect methods for estimating the hydraulic properties of unsaturated soils. Univ. of California, Riverside.

Basile, A., and G. D'Urso. 1997. Experimental corrections of simplified methods for predicting water retention curves in clay-loamy soils from particle-size determination. Soil Technol. 10:261–272.

Buczko, U., and H.H. Gerke. 2005. Evaluation of the Arya–Paris model for estimating water retention characteristics of lignitic mine soils. Soil Sci. 170(7):483–494.

Chan, T.P., and R.S. Govindaraju. 2004. Estimating soil water retention curve

from particle-size distribution data based on polydisperse sphere systems. Vadose Zone J. 3:1443–1454.

Haverkamp, R., and J.Y. Parlange. 1982. Comments on "A physicoempirical model to predict the soil moisture characteristic from particle-size distribution and bulk density data.". Soil Sci. Soc. Am. J. 46:1348.

Haverkamp, R., and J.Y. Parlange. 1986. Predicting the water-retention curve from particle-size distribution: 1. Sandy soils without organic matter. Soil Sci. 142:325–339.

Haverkamp, R., P. Reggiani, and J.R. Nimmo. 2002. Property-transfer models. p. 759–761, 781–782. *In* J.H. Dane and G.C. Topp (ed.) Methods of soil analysis: Part 4. Physical methods. SSSA, Madison, WI.

Hoffmann-Riem, H., M.Th. van Genuchten, and H. Flühler. 1999. General model of the hydraulic conductivity of unsaturated soils. p. 31–42. *In* M.Th. van Genuchten and F.J. Leij (ed.) Proc. of the Int. Workshop on Characterization and Measurement of the Hydraulic Properties of Unsaturated Porous Media. Univ. of California, Riverside.

Hwang, S.I., and S.E. Powers. 2003. Using particle-size distribution models to estimate soil hydraulic properties. Soil Sci. Soc. Am. J. 67:1103–1112.

Kosugi, K. 1999. General model for unsaturated hydraulic conductivity for soils with lognormal pore-size distribution. Soil Sci. Soc. Am. J. 63:270–277.

Mishra, S., J. Parker, and N. Singhal. 1989. Estimation of soil hydraulic properties and their uncertainty from particle size distribution data. J. Hydrol. 108:1–18.

Mualem, Y. 1976. A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resour. Res. 12:513–522.

Mualem, Y., and G. Dagan. 1978. Hydraulic conductivity of soils: Unified approach to the statistical models. Soil Sci. Soc. Am. J. 42:392–395.

Nimmo, J.R. 1997. Modeling structural influences on soil water retention. Soil Sci. Soc. Am. J. 61:712–719.

Ralston, A. 1965. A first course in numerical analysis. McGraw-Hill, New York.

Schaap, M.G., F.J. Leij, and M.Th. van Genuchten. 2001. Rosetta: A computer program for estimating soil hydraulic parameters with hierarchical pedotransfer functions. J. Hydrol. 251:163–176.

Soil Conservation Service–USDA. 1974. Soil survey laboratory data and descriptions for some soils of New Jersey. Soil Survey Investigations Rep. 26. New Jersey Agricultural Experiment Station, Rutgers Univ., New Brunswick, NJ.

Vaz, C.M.P., M. de Freitas Iossi, J. de Mendonca Naime, A. Macedo, J.M. Reichert, D.J. Reinert, and M. Cooper. 2005. Validation of the Arya and Paris water retention model for Brazilian soils. Soil Sci. Soc. Am. J. 69:577–583.

Winfield, K.A. 2005. Development of property-transfer models for estimating the hydraulic properties of deep sediments at the Idaho National Engineering and Environmental Laboratory, Idaho. Scientific Investigations Rep. 2005-5114. Available at http://pubs.water.usgs.gov/sir20055114/ (verified 28 June 2007). USGS, Idaho Falls.

Winfield, K.A., J.R. Nimmo, J.A. Izbicki, and P.M. Martin. 2006. Resolving structural influences on water-retention properties of alluvial deposits. Vadose Zone J. 5:706–719.